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*“Sunspots, cycles and adjustment costs in the two-sectors model”*

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# Sunspots, cycles and adjustment costs in the two-sectors model

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**Abstract:** *Do adjustment costs able to modify the dynamic of the two sectors model? We examine the impact of adjustment costs in capital on the properties of long-run equilibrium. We propose to analyse how the positive and negative degrees of adjustment costs could interplay with the local indeterminacy mechanism coming from the presence of sector specific externalities. When the adjustments costs function is convex there exists a Höpf bifurcation and the trajectory describes a cycle around the steady state. We give an heuristic economic explanation of the role of the adjustment costs leading to economic cycles.*

**Keywords:** *Two-sector growth model, externality, adjustment costs, endogenous fluctuations.*

JEL classification: C62, E32, 041

## 1 Introduction

Many interesting findings have emerged from the study of indeterminacy and endogenous fluctuations, i.e. the existence of a continuum of equilibria that arises in dynamic economies with some market imperfections. In a major contribution, Benhabib and Nishimura [7] examined the two-sector model with different Cobb-Douglas technologies at the private level with sector-specific externalities and constant social returns to scale. They prove, with a separable utility function which is linear in consumption and strictly concave with respect to labor, that local indeterminacy arises if and only if technological externalities allow factor intensities between the private and social levels to be reversed (i.e. the consumption good is capital intensive at the private level and labor intensive at the social level). Consequently, there is a technological mechanism arising from externalities which breaks the duality between the Rybczynsky and Stolper-Samuelson effects and leads to

indeterminacy. Garnier, Venditti and Nishimura [11] subsequently introduce non-linear utility in consumption. Under factor intensity reversal between private and social levels, they prove that sunspot fluctuations exist if and only if the elasticity of intertemporal substitution in consumption is high enough and the elasticity of labor supply is low enough (even equal to zero).

Given these findings, it appears relevant to ask whether introducing an adjustment cost in capital might affect dynamic of the model. The idea that the installation of a new capital could generate additional costs (positive or negative) is widely viewed as an important feature of the investment decision analysis. Neoclassical studies of investment behavior often ignore variations in capacity utilization.

This paper shows how investment adjustment costs interact with positive sector specific externalities in the two-sector model à la Benhabib and Nishimura [7]. In the one sector model, the introduction of adjustment costs make it difficult for indeterminacy to occur: the required degree of increasing returns is higher in the presence of such costs as in the paper of Jinill Kim [15]. Nevertheless, in the two sector model with constant returns to scale at the social level and decreasing at the private level, the indeterminacy mechanism is different and the presence of adjustment costs interacts positively with the indeterminacy mechanism. In this way, we show that the presence of such costs make it possible the Höpf bifurcation in the standard two-sector model (with only positive sector specific externalities, exogenous labor and linear utility function in consumption) whereas it's not possible otherwise. The unique contribution of this paper is to present this relationship in a simple analytic way and to prove the existence of the Höpf bifurcation.

The rest of the paper is organized as follows. Section 2 describes the economy. Section 3 characterizes the competitive equilibrium. Section 4 analyzes the mechanism that leads to equilibrium indeterminacy. Section 5 gives an example of utility function that allows the existence of indeterminacy and illustrates our main result through a standard parametrization of the model. Section 6 concludes. All the proofs are collected in the appendix.

## 2 The economy

We consider an infinite horizon, continuous time, two-sector model with Cobb-Douglas technologies, inelastic labor supply and non linear utility function in consumption. The economy consists of competitive firms and a representative household.

### 2.1 Firms

We assume that consumption good  $y_0$  and capital good  $y_1$  are produced by capital  $x_{1j}$  and labor  $x_{0j}$ ,  $j = 0, 1$ , through a Cobb-Douglas technology which contains sector specific externalities  $e_j$ . The representative firm in each industry faces the following technology called *private production function*:

$$y_j = F_j(x_{0j}, x_{1j}) = x_{0j}^{\beta_{0j}} x_{1j}^{\beta_{1j}} e_j(X_{0j}, X_{1j}) \quad \text{for } j = 0, 1 \quad (1)$$

with  $\beta_{ij} \in [0, 1]$  and  $X_{ij}$  the average use of input  $i$  in the sector  $j$ .

The positive sector externalities are such that:

$$e_j(X_{0j}, X_{1j}) = X_{0j}^{b_{0j}} X_{1j}^{b_{1j}} \quad (2)$$

We assume that this economy wide average are taken as given by each individual firms. At the equilibrium, since all firms of sector  $j$  are identical, we have  $X_{ij} = x_{ij}$  and we may define the *social production function* as follows:

$$y_j = x_{0j}^{\hat{\beta}_{0j}} x_{1j}^{\hat{\beta}_{1j}} \quad \text{for } j = 0, 1 \quad (3)$$

with  $\hat{\beta}_{ij} = \beta_{ij} + b_{ij}$  we assume that the returns to scale are constant at the social level and decreasing at the private level: in each sector  $j = 0, 1$ ,  $\hat{\beta}_{0j} + \hat{\beta}_{1j} = 1$ .

The labor is exogenous, therefore the total labor, normalized to one, is given by:

$$x_{00} + x_{01} = 1 \quad (4)$$

and the total stock of capital is given by:

$$x_{10} + x_{11} = x_1 \quad (5)$$

Choosing the consumption good as the numeraire, i.e.  $p_0 = 1$ , a firm in each industry maximizes its profit given the output price of the investment  $p_1$ , the rental rate of capital  $w_1$  and the wage rate  $w_0$ . The first order conditions subject to the private technologies (1) give

$$x_{ij}/y_j = p_j \beta_{ij}/w_i \equiv a_{ij}(w_i, p_j), \quad i, j = 0, 1 \quad (6)$$

We call  $a_{ij}$  the input coefficients from the *private* viewpoint. If the agents take account of externalities as endogenous variables in profit maximization, the first order conditions subject to the social technologies (3) give on the contrary

$$x_{ij}/y_j = p_j \hat{\beta}_{ij}/w_i \equiv \hat{a}_{ij}(w_i, p_j), \quad i, j = 0, 1 \quad (7)$$

We call  $\hat{a}_{ij}$  the input coefficients from the *social* viewpoint. As we will show below, the factor-price frontier, which gives a relationship between input prices and output prices, is expressed with the input coefficients from the social viewpoint.

Based on these input coefficients it may be shown that the factor-price frontier is determined by the input coefficients from the social viewpoint while the factor market clearing equation depends on the input coefficients from the private perspective:\*

**Lemma 1** : Denote  $p = (1, p_1)'$ ,  $w = (w_0, w_1)'$  and  $\hat{A}(w, p) = [\hat{a}_{ij}(w_i, p_j)]$ . Then  $p = \hat{A}'(w, p)w$ .

**Lemma 2** : Denote  $x = (1, x_1)'$ ,  $y = (y_0, y_1)'$  and  $A(w, p) = [a_{ij}(w_i, p_j)]$ . Then  $A(w, p)y = x$ .

Note that at the equilibrium, the rental rate is function of the output price only, i.e.  $w_1 = w_1(p_1)$ , while outputs are functions of the capital stock, total labor and the output price,  $y_j = \tilde{y}_j(x_1, p_1)$ ,  $j = 0, 1$ .

## 2.2 Household

We assume that the population is constant and normalized to one. At the date  $t$ , the representative agent derives his utility  $U(\cdot)$  from consumption

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\*See Garnier, Nishimura and Venditti [10] for the proofs of these results.

$c(t)$ . Considering the external effects as given, profit maximization in both sector gives demand functions as function of capital stock  $x_1(t)$ , production level of the investment good  $y_1(t)$  and external effects  $(e_0, e_1)$ , namely  $\tilde{x}_{ij} = x_{ij}(x_1, y_1, e_0, e_1)$  for  $i, j = 0, 1$ . The production frontier is then defined as:

$$y_0 = T(x_1, y_1, e_0, e_1) = \underset{\tilde{x}_{ij}}{Max} \tilde{x}_{00}^{\beta_{00}} \tilde{x}_{10}^{\beta_{10}} e_0$$

s.t. (1)            (3)            (4)

From the envelop we get:  $\frac{\partial T}{\partial x_1} = w_1$  and  $\frac{\partial T}{\partial y_1} = -p_1$ . In this model, the representative agent consumes the whole consumption good therefore we have  $c = y_0$  so he solves the following intertemporal maximization problem<sup>†</sup>:

$$\begin{aligned} & \underset{y_1(t), x_1(t)}{max} \int_0^\infty e^{-\rho t} c(t) dt && \text{s.c.} \\ \text{s.c.} & \dot{x}(t) = x_1(t) \Phi \left( \frac{y_1(t)}{x_1(t)} \right) && (8) \\ & x_1(0) = x_1 \text{ and } \{e_0(t), e_1(t)\}_{t \geq 0} \text{ given} \end{aligned}$$

Where  $\rho > 0$  is the subjective discount rate and  $\Phi$  the function of investment adjustment costs. We incorporate the investment adjustment costs in the capital accumulation equation in a way similar to Lucas and Prescott [16]. In this specification, the adjustment costs occur when the level of capital stock changes. We note that the classical expression of capital accumulation corresponds to the particular adjustment costs function  $\Phi \left( \frac{y_1}{x_1} \right) = \frac{y_1}{x_1} - g$  with  $g$  the constant depreciation rate of capital. The adjustment costs could be thought as a measure of the efficiency of the investment i.e. efficiency index of the investment. For example, in the extrem cases, if the investment is so efficient, an investment per capital unit  $\frac{y_1}{x_1} < 1$  gives an efficiency index  $\Phi \left( \frac{y_1}{x_1} \right) > 1$  and the capital stock increases (the adjustments costs are negative) and if it is so inefficient an investment per capital unit  $\frac{y_1}{x_1} > 1$  gives an efficiency index  $\Phi \left( \frac{y_1}{x_1} \right) < 1$  and the capital stock decreases (the adjustments costs are positive).

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<sup>†</sup>We suppose that the utility function is linear in consumption: Garnier, Venditti and Nishimura[11] have shown that the parameter preference have to be small (close to 0) to allow indeterminacy in the two sector model with sector specific externalities.



For the later analysis of the local dynamics, we make two assumptions on the specific form of the adjustment costs function.

**Assumption 1** : *The adjustment costs function satisfies:*

1.  $\Phi(g) = 0$
2.  $\Phi'(g) = 1$

The first assumption defines the depreciation rate,  $g$ , as the ratio between investment and capital at the steady state. The second assumption makes the steady state of our model with adjustment costs the same that the one with linear capital accumulation equation. We don't impose convexity or concavity of the function of adjustment costs, consequently  $\Phi'(g) > 0$  or  $\Phi'(g) < 0$ .

The Hamiltonian in current value of (8) is:

$$H = T(x_1, y_1) + q_1(t) \left( x_1(t) \Phi \left( \frac{y_1(t)}{x_1(t)} \right) \right) \quad (9)$$

The first order conditions are

$$p_1(t) = q_1(t) \Phi' \left( \frac{y_1(t)}{x_1(t)} \right) \quad (10)$$

$$\dot{q}_1(t) = q_1(t) \left[ \rho - \Phi \left( \frac{y_1(t)}{x_1(t)} \right) + \frac{y_1(t)}{x_1(t)} \Phi' \left( \frac{y_1(t)}{x_1(t)} \right) \right] - w_1 \quad (11)$$

$$\dot{x}_1 = x_1 \Phi \left( \frac{y_1(t)}{x_1(t)} \right) \quad (12)$$

with the transversality condition:

$$\lim_{t \rightarrow +\infty} x_1(t) p_1(t) e^{-\rho \cdot t} = 0 \quad (13)$$

Where  $q_1$  is the co-state variable which corresponds to the utility price of capital in current value.

### 3 The competitive equilibrium

We use the parameter  $\varphi = -\frac{y_1 \Phi''}{x_1 \Phi'}$  to express the degree of the investment adjustment costs i.e. the elasticity of the investment adjustment costs. Therefore,  $\varphi$  could be seen as a measure of the efficiency index of the investment per capita.

To obtain the dynamic equations characterizing the equilibrium path, we combine (10) and (11) (after a total differentiation of (10)) and we obtain two equations of motion which describe the dynamic of equilibrium paths<sup>‡</sup>

$$\dot{x}_1 = x_1 \Phi \quad (14)$$

$$\dot{p}_1 = \frac{1}{E} \left[ p_1 \left( \rho + \frac{y_1}{p_1} \Phi' - \Phi \right) - \Phi' w_1 + \varphi \frac{p_1}{y_1} \left( \frac{\partial y_1}{\partial x_1} - \frac{y_1}{x_1} \right) x_1 \right] \quad (15)$$

with  $E = 1 + \varphi \frac{p_1}{y_1} \frac{\partial y_1}{\partial p_1}$ .

Any solution  $\{x_1(t), p_1(t)\}_{t \geq 0}$  of the system (14) satisfying the transversality condition (13) will be called equilibrium path.

#### 3.1 Steady state

We want to study the dynamical system (14) in the neighborhood of the steady state.

**Proposition 1** *Under assumption 1 there exists a unique steady state  $(x_1^*, p_1^*) > 0$  solution of :*

$$\begin{aligned} \dot{x}_1 = 0 &\iff y_1(x_1, p_1) = g x_1 \\ \dot{p}_1 = 0 &\iff w_1(p_1) = p_1(\delta + g) \end{aligned}$$

We note that the steady state is the same that the one of the model with linear capital accumulation.

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<sup>‡</sup>We note that all function depends on  $x_1$  and  $p_1$  :  $w_1 = w_1(p_1)$ ,  $y_1 = y_1(x_1, p_1)$  and  $\Phi = \Phi\left(\frac{y_1(x_1, p_1)}{x_1}\right)$

### 3.2 The linearized system

In order to study the indeterminacy properties of equilibrium, we linearize the system (14) around  $(x_1^*, p_1^*)$  which gives the following Jacobian matrix<sup>§</sup>:

$$J = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} - g & \frac{\partial y_1}{\partial p_1} - g \\ \frac{\varphi^* \frac{p_1}{y_1} \left( \frac{\partial y_1}{\partial x_1} - g \right) \left( g + \rho - \frac{\partial y_1}{\partial x_1} \right)}{E^*} & \left\{ \delta + g - \frac{\partial w_1}{\partial p_1} - \varphi^* \frac{p_1}{y_1} \left[ \left( \frac{\partial y_1}{\partial x_1} - g \right) \left( \frac{\partial y_1}{\partial p_1} - g \right) - \rho \frac{\partial y_1}{\partial p_1} \right] \right\} \frac{\partial y_1}{\partial p_1} - g \end{pmatrix} \quad (16)$$

Given initial capital stock  $x_1(0)$  if there is more than one initial price  $p_1(0)$  in the stable manifold of  $(x_1^*, p_1^*)$ , the equilibrium path coming from  $x_1(0)$  will not be unique. In particular, if the Jacobian matrix  $J$  (16) has two eigenvalues with negative real part (the locally stable manifold of the steady state  $(x_1^*, p_1^*)$  is two dimensional), there will be a continuum of converging paths and thus a continuum of equilibria:  $(x_1^*, p_1^*)$  is said to be locally indeterminate.

The dynamics of the model around the steady state can be fully derived from the eigenvalues of Jacobian matrix (16). If we denote  $T$  and  $D$  the trace and the determinant of the Jacobian matrix (16), we know that the steady state is locally indeterminate if and only if  $T < 0$  et  $D > 0$ . Therefore, we need to study the sign of  $T$  and  $D$  given by:

$$T = \frac{1}{E^*} \left\{ \frac{\partial y_1}{\partial x_1} - \frac{\partial w_1}{\partial p_1} - \rho + \varphi^* \frac{p_1}{y_1} \left[ g \left( \frac{\partial y_1}{\partial x_1} - g \right) + \rho \frac{\partial y_1}{\partial p_1} \right] \right\} \quad (17)$$

$$D = \frac{1}{E^*} \left( \frac{\partial y_1}{\partial x_1} - g \right) \left( \rho + g - \frac{\partial w_1}{\partial p_1} + \varphi^* g \rho \frac{p_1}{y_1} \right) \quad (18)$$

## 4 Existence of local indeterminacy

Our main objective is to study the impact of adjustment costs measured by the elasticity  $\varphi^*$  on the indeterminacy mechanism coming from sector

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<sup>§</sup>At the steady state, under the assumption 1 the elasticity  $\varphi$  becomes:

$$\varphi^* = -g\Phi''(g) \geq 0$$

specific externalities.

Solving the system (17-18) with respect to  $\varphi^*$  gives a linear relationship between  $T(\varphi^*)$  and  $D(\varphi^*)$ : when  $\varphi^*$  varies on  $]-\infty, +\infty[$ ,  $T(\varphi^*)$  and  $D(\varphi^*)$  move along the line called in what follows  $\Delta_\varphi$ , which is defined by<sup>¶</sup>:

$$D = S_\varphi T + M_\varphi$$

with

$$\begin{aligned} S_\varphi &= \frac{\left(\frac{\partial y_1}{\partial x_1} - g\right) \left[\frac{\partial y_1}{\partial p_1} \left(\rho + g - \frac{\partial w_1}{\partial p_1}\right) + \rho g\right]}{g \left(\frac{\partial y_1}{\partial x_1} - g\right) + \rho \frac{\partial y_1}{\partial p_1} + \frac{\partial y_1}{\partial p_1} \left(\rho + g - \frac{\partial w_1}{\partial p_1}\right)} \\ M_\varphi &= \frac{\left[g \left(\frac{\partial y_1}{\partial x_1} - g\right) + \rho \frac{\partial y_1}{\partial p_1}\right] \left(\frac{\partial y_1}{\partial x_1} - g\right) \left(\rho + g - \frac{\partial w_1}{\partial p_1}\right) - \rho g \left(\frac{\partial y_1}{\partial x_1} - g\right) \left(\rho + g - \frac{\partial w_1}{\partial p_1}\right)}{g \left(\frac{\partial y_1}{\partial x_1} - g\right) + \rho \frac{\partial y_1}{\partial p_1} + \frac{\partial y_1}{\partial p_1} \left(\rho + g - \frac{\partial w_1}{\partial p_1}\right)} \end{aligned} \quad (19)$$

Note that  $S_\varphi$  and  $M_\varphi$  depend only upon technological parameters.

We use the geometrical method of Grandmont, Pintus and De Vilder [12] in order to study the variations of  $T(\varphi)$  and  $D(\varphi)$  in the  $(T, D)$  plane, when  $\varphi^*$  varies continuously on  $]-\infty, +\infty[$ .

#### 4.1 Condition for local indeterminacy without adjustment costs i.e. $\varphi^* = 0$

In the case of  $\varphi^* = 0$  there is no adjustment costs what it's correspond to the linear accumulation of capital, we get  $E^* = 1$  and:

$$T(0) = \frac{\partial y_1}{\partial x_1} - \frac{\partial w_1}{\partial p_1} - \rho \text{ and } D(0) = \left(\frac{\partial y_1}{\partial x_1} - g\right) \left(\rho + g - \frac{\partial w_1}{\partial p_1}\right)$$

with:

$$\frac{\partial y_1}{\partial x_1} = \frac{a_{00}}{a_{00}a_{11} - a_{01}a_{10}} \text{ and } \frac{\partial w_1}{\partial p_1} = \frac{\hat{a}_{00}}{\hat{a}_{00}\hat{a}_{11} - \hat{a}_{01}\hat{a}_{10}}$$

We note that  $\frac{\partial y_1}{\partial x_1}$  represents the Rybczynsky effect (i.e. quantity effect) and  $\frac{\partial w_1}{\partial p_1}$  the Stolper Samuelson effect (i.e. price effect).

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<sup>¶</sup>Note that  $(x_1^*, p_1^*)$  does not depend on  $\varphi^*$  and remains the same along line  $\Delta_\alpha$ .

We can characterize both partial derivatives in terms of capital intensity differences across private and social levels as in Benhabib and Nishimura [7]. Using the input coefficients given 6 and 7, we give the following definition:

**Definition 1** : *The consumption good is said to be:*

i) *capital (labor) intensive at the private level if and only if:*

$$a_{00}a_{11} - a_{01}a_{10} < (>)0$$

ii) *capital (labor) intensive at the social level if and only if:*

$$\hat{a}_{00}\hat{a}_{11} - \hat{a}_{10}\hat{a}_{01} < (>)0$$

At the steady state, it's possible to give these condition i) and ii) only with the technological parameters  $\beta_{ij}$  and  $\hat{\beta}_{ij}$ .

**Proposition 2** : *Let  $b \equiv \beta_{00}\beta_{11} - \beta_{01}\beta_{10}$  and  $\hat{b} \equiv \hat{\beta}_{00}\hat{\beta}_{11} - \hat{\beta}_{10}\hat{\beta}_{01}$ . At the steady state we have:*

i)  $a_{00}a_{11} - a_{01}a_{10} < (>)0 \Leftrightarrow b < (>)0$

ii)  $\hat{a}_{00}\hat{a}_{11} - \hat{a}_{10}\hat{a}_{01} < (>)0 \Leftrightarrow \hat{b} < (>)0$

It follows that  $\partial y_1 / \partial x_1$  corresponds to the factor intensity difference from the private viewpoint (Rybczynski effects), while  $\partial w_1 / \partial p_1$  corresponds to the factor intensity difference from the social viewpoint (Stolper-Samuelson effects). Therefore, we can give the indeterminacy condition given by Benhabib and Nishimura [7] in the two-sectors model with exogenous labor, linear utility function, sector specific externalities and linear capital accumulation:

**Proposition 3** : *The steady state is locally indeterminate if and only if the consumption good is capital intensive at the private level ( $b < 0$ ) and labor intensive at the social level ( $\hat{b} > 0$ ) i.e. there is a factor intensities reversal between the private and the social perspective.*

This factor intensities reverseal corresponds to a break of the duality between Rybczynszy and Stolper Samuelson effects.

With the proposition 3 we know that the  $\Delta_\varphi$  line reaches the local indeterminacy area of  $(T, D)$  plane when  $\varphi^* = 0$ , if and only if  $b < 0$  and  $\hat{b} > 0$ . Therefore, the presence of adjustment costs only is not sufficient to lead to local indeterminacy without sector specific externalities but we will see they can interplay with externalities to provide Höpf bifurcation.

Now, to ensure we have the steady state locally indeterminate when  $\varphi^* = 0$ , we have to make the following assumption:

**Assumption 2** :  $b < 0$  and  $\hat{b} > 0$ .

## 4.2 Infinite degree of adjustment costs i.e. $\varphi^* \rightarrow \pm\infty$

In the case of infinite degree of adjustment costs i.e.  $\varphi^* \rightarrow \pm\infty$  the Trace and the Determinant (17,18) become:

$$T(\infty) = g \frac{\left(\frac{\partial y_1}{\partial x_1} - g\right)}{\frac{\partial y_1}{\partial p_1}} + \rho$$

$$D(\infty) = g\rho \frac{\left(\frac{\partial y_1}{\partial x_1} - g\right)}{\frac{\partial y_1}{\partial p_1}}$$

with:

$$\frac{\partial y_1}{\partial p_1} = \frac{\partial y_1}{\partial x_1} \frac{1}{p_1} \left[ x_1 \left( 1 - \frac{\hat{\beta}_{01}}{\hat{\beta}_{00}} \right)^{-1} + \frac{a_{10}}{a_{00}} \left( 1 - \frac{\hat{\beta}_{11}}{\hat{\beta}_{10}} \right)^{-1} \right] - \frac{y_1}{p_1}$$

Consequently, it's possible to express  $D(\infty)$  as linear function of  $T(\infty)$  that we note  $\Delta_\infty$ :

$$D(\infty) = \rho T(\infty) - \rho^2$$

This line  $\Delta_\infty$  represents both the start points and the end points set of the segment  $\Delta_\varphi$ . Therefore, we can see immediatly that this line  $\Delta_\infty$  can not get throught the indeterminacy area of the  $(T, D)$ plane for any values of  $b$  and  $\hat{b}$  since both  $D(+\infty)$  and  $T(+\infty)$  have always the same sign (positive or negative) for small values of the parameter  $\rho$ . This sign is rely on the sign

of the both derivatives  $\frac{\partial y_1}{\partial x_1}$  (which depends on the sign of the parameter  $b$ ) and  $\frac{\partial y_1}{\partial p_1}$ .

Now, we focus on the case characterized by the assumption 2. Indeed, on this assumption, we have the pair  $(T(0), D(0))$  in the indeterminacy area of the  $(T, D)$  plane. Consequently, if we can verify that  $T(\infty) > 0$  and  $D(\infty) > 0$  then we know that the line  $\Delta\varphi$  cuts the D-axes that is the trace of the Jacobian matrix is nul and we have a Höpf bifurcation (i.e. cycles exist).

**Proposition 4** : *On the assumption 2,  $T(\infty) > 0$  and  $D(\infty) > 0$  if and only if  $\hat{\beta}_{01} < \hat{\beta}_{00}$ .*

We make the following assumption:

**Assumption 3** :  $\hat{\beta}_{01} < \hat{\beta}_{00}$ .

This assumption ensures that the derivative  $\frac{\partial y_1}{\partial p_1}$  is negative when assumption 2 is verified.

### 4.3 General case: finite degree of adjustment costs i.e. $\varphi^* > 0$

Finally, We have to study the way of move of the pair  $(T(\varphi), D(\varphi))$  on the line  $\Delta\varphi$ . It sufficients to compute the derivative  $\frac{\partial D}{\partial \varphi}$ <sup>||</sup>:

$$\frac{\partial D}{\partial \varphi} = \frac{\frac{P_1}{y_1} \left( \frac{\partial y_1}{\partial x_1} - g \right) \left[ g\rho - \frac{\partial y_1}{\partial p_1} \left( \rho + g - \frac{\partial w_1}{\partial p_1} \right) \right]}{E^2}$$

On assumptions 2 and 3, it's easy to check that  $\frac{\partial D}{\partial \varphi} > 0$ . Conséquently, when  $\varphi^*$  increases from  $-\infty$  to  $+\infty$ , the pair  $(T(\varphi), D(\varphi))$  increases along the line  $\Delta\varphi$  from the starting point such that  $T(\infty) > 0$  and  $D(\infty) > 0$ , cuts the D-axes, gets through the point  $(T(0), D(0))$ , gets out of the indeterminacy area and returns on the starting point from below. we can give the following proposition:

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<sup>||</sup>Indeed, we have the starting point, the middle point  $(T(0), D(0))$  and the end point therefore if we know only the sign of the derivative  $\frac{\partial D}{\partial \varphi}$  we know the way of move of the pair  $(T(\varphi), D(\varphi))$  on the line  $\Delta\varphi$ .

**Proposition 5** : *On the assumption 1, 2 and 3  $\exists \varphi^c < 0$  and  $\bar{\varphi} > 0$  such that:*

- i)  $T(\varphi^c) = 0$  and  $D(\varphi^c) > 0$
- ii)  $\forall \varphi^* \in ]\varphi^c, \bar{\varphi}[ : T(\varphi^*) < 0$  and  $D(\varphi^*) > 0$ .

The case i) ensures the existence of the value  $\varphi^c < 0$  such that we have a Höpf bifurcation: the steady state is a center and there exists periodic stable trajectory. This case is possible for a negative degree of adjustment costs only i.e.  $\Phi$  is convex adjustment costs function. The presence of convex adjustment costs does not allow the firm to make instantaneous changes in the stock of capital when the price of investment good is modified since this adjustment of capital will have an infinite cost. Consequently, the firm have to adjust progressively the stock of capital.

The case ii) explains the local indeterminacy of the steady state is possible for an interval of values of adjustment costs degree which includes 0. If the negative degree of adjustment costs is too large (i.e.  $0 < \varphi^c < \varphi^*$ ) the steady state becomes instable and if it the positive degree of adjustment costs is too large (i.e.  $\varphi^* > \bar{\varphi} > 0$ ) the steady state is a saddle point stable.

Now, we try to explain the economic intuition of impact of the adjustment costs on the indeterminacy mechanism coming from factor intensities reverseal between the private and social level that is what is the role of adjustment costs in the trajectories switching which are the source of sunspots.

Starting from an arbitrary equilibrium, consider that the agent expects another equilibrium with a larger rate of investment due to an instantaneous increase in the relative price of investment good  $p_1$ . The only way for this other equilibrium path to become a new equilibrium path is to find a mechanism which reverses the price toward the equilibrium and offsets this initial increase. But the rise in the stock of capital, due to a higher rate of investment depends on the adjustment costs function. If the adjustment costs function is convex (resp. concave) i.e.  $\varphi^* < 0$  (resp.  $\varphi^* > 0$ ), the stock of capital will increase more than proportional (resp. it will increase less than proportional) for a low increase of investment. Consequently, from the Rybczynsky theorem ( $\frac{\partial y_1}{\partial x_1} < 0$ ), since the investment good is labor intensive at



the private level ( $b < 0$ ) i.e. the consumption good is capital intensive at the private level, there is a more than proportional decrease in its output<sup>\*\*</sup>. This more than proportional decrease is amplified (resp. diminished) by the adjustments costs through the variation of the capital stock. Moreover, from the Stolper Samuelson effect ( $\frac{\partial w_1}{\partial p_1} > 0$ ), since the investment good is capital intensive at the social level ( $\hat{b} > 0$ ), a rise in initial price of the investment good leads to an increase in the rate of return of capital. This increase have to be offset by a new decrease in the investment price to maintain the overall return of capital such that<sup>††</sup>:

$$\frac{\dot{p}_1}{p_1} + \frac{w_1}{p_1} = \rho + g + \varphi * \left( \frac{\partial y_1}{\partial x_1} - g \right) \dot{x}_1$$

The amplitude of this new decrease depends on the degree of adjustment costs: it can be offset the initial rise i.e  $\varphi * > 0$  but it can be also greater than the initial rise and leads to another fluctuations i.e.  $\varphi * < 0$ . There is a limit case where this amplitude leads to reverse mechanism where the variables take exactly the opposite values that is we have a trajectory which describes a cycle i.e  $\varphi * = \varphi^c$ .

## 5 Concluding comments

In this paper we have prove that convex adjustment costs function interplays with sector specific externalities to lead to Höpf bifurcation and then periodic cycles.

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<sup>\*\*</sup>This decrease is the key of the indeterminacy mechanism coming from sector specific externalities which break the duality between Rybczynsky effects and Stolper Samuelson effects (Benhabib and Nishimura [7]).

<sup>††</sup>From the equation 12.

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