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Abstract : *Our main objective is to study the impact of consumption externality like keeping of with the Joneses on the properties of long-run equilibrium in the two-sector optimal growth model. Does this consumption externality lead to a new mechanism of local indeterminacy and endogenous fluctuations ? We will see that, in two-sector growth models with exogenous labor and without technological externalities, if the representative agent is able to give more value to his social status than his own consumption, this is the keys of a new mechanism for endogenous fluctuations. Moreover, by opposition with the other endogenous fluctuation mechanisms, we will see that this one doesn't need to have restriction on the factor intensity configuration of the consumption sector.*

Keywords : *Two-sector models, continous-time models, consumption externality, keeping up with the Joneses, local indeterminacy, endogenous fluctuations.*

1 Introduction

The study of endogenous fluctuations, i.e. the existence of a continuum of equilibria that arises in dynamic economies with some market imperfections, have given a lot of results. In general equilibrium model, the existence of a continuum of equilibria which converges to the same steady state is relied to the local indeterminacy property of the steady state. Generally, indeterminacy emerges with small market distortions as a type of coordination problem. Following Benhabib and Nishimura[7], these distortions lead to a mechanism such that, starting from an arbitrary equilibrium, if all agents were simultaneously increse their investment in an asset, the rate of return on the asset would tend to increase, and in turn set off relative price changes that would drive the economy back towards steady-state. In the one-sector models, increasing returns, via external effects on production or monopolistic competition, are able to generate this mechanism. In the two-sector models, the rate of return and marginal products depend not only on stocks of assets, but also on the composition of output across sectors. Increasing the production and the stock on a capital, say due to an increase in its price, may well increase its rate of return. It is possible therefore to have constant aggregate returns in all sectors at the social level (but decreasing at the private level) and to still obtain indeterminacy. For that, it has to have external effect, through sector specific externalities, at least in one of the sectors. Consequently, the major contributions of Benhabib and Nishimura [7] and Benhabib, Nishimura and Venditti [8], in the two-sector model with different Cobb-Douglas technologies at the private level with sector-specific externalities and constant social returns to scale, Benhabib and Nishimura [7] prove, with a separable utility function which is linear in consumption and strictly concave with respect to labor, that local indeterminacy arises if and only if technological externalities allow to have a reversal of factor intensities between the private and social levels (i.e. the consumption good is capital intensive at the private level and labor intensive at the social level). Therefore, there exists technological mechanism coming from externalities which brokes the duality between Rybczynsky and Stolper-Samuelson effects and leads to indeterminacy. In all these contributions, an implication of decreasing private returns is positive profits. The presence of such profits (even quite small) would invite entry, and unless the number of firms cannot be constant along the equilibrium and a fixed cost of entry has to be assumed to solve this problem, but it is not really satisfying.

More recently, Nishimura, Takahashi and Venditti [18] have proved that indeterminacy is possible without any market distortions, so in the two-sectors optimal growth discret time models with CES technologies, non linear preferences are able to generate a mechanism such that for high enough values of elasticity of intertemporal substitution in consumption, indeterminacy is led, but only if the consumption good is capital intensive. In this case, this is the intertemporal arbitrage behavior of the representative agent and his capacity to substitute future consumption to the present one that is the source of the endogenous fluctuations. Indeed, oscillations in the consumption and investment goods output, led by fluctuations, require some oscillations in the consumption levels, that is the case when the elasticity of intertemporal substitution in consumption is high enough (i.e. the degree of concavity of the utility function is low enough). In this line, Garnier, Venditti and Nishimura [12] have introduced non linear utility in consumption in the two-sector continous-time model with sector specific externalities such that there are constant returns to scale at the social level but decreasing at the private level. With the breaking of Rybczynsky and Stolper-Samuelson effects (like in Benhabib and Nishimura [7] and Benhabib, Nishimura and Venditti [8]), they prove the existence of sunspot fluctuations is obtained if and only if the elasticity of intertemporal substitution in consumption is large enough and the elasticity of labor supply is low enough (even equal to zero). Therefore, the role played by non linear preferences seems the same than the one of Nishimura, Takahashi and Venditti [18], that is, it allows the representative agent to smooth his consumption over time.

In all these contributions, a necessary condition to obtain indeterminacy is the presence of capital intensive consumption good since the coordination problem needs that all agents increase their investment in the capital asset what leads to a first increase (before the future decreases coming from the fluctuations) of the more capital intensive good : the consumption. Whereas the output of the consumption good increases, the consumption of agents too. Then, even if this rise of the consumption will be followed by future decreases, the representative agent has a present time preference and is able to smooth his consumption over time. For these reasons, this indeterminacy mechanism is accepted by the agents. The major problem of the existence of capital intensive consumption good is that the capital share in the production function has to be greater than the labor share, but we know that this fact is not consistent empirically. Moreover, whereas the consumption good is capital intensive, then the investment good has to be labor intensive.

In order to show that it is possible to have a new mechanism of endogenous fluctuations led only by the preferences and that, even if the consumption good is not capital intensive, we introduce a new type of externality called keeping up with the Joneses. This consumption externality operates through the utility function by adding an exogenous variable that represents the consumption standard of the economy. The idea that the happiness of an individual depends upon the consumption of others is widely viewed as an important feature of our shared social existence. The introduction of consumption standard in the utility function generates an externality which increases the felicity that each individual obtains from his own consumption (i.e. the utility function is increasing according to the consumption standard). Preferences exhibit "keeping up with the Joneses" since we assume that the marginal utility of consumption is increasing with respect to the externalities.

Liu and Turnovsky [17] have showed that the consumption externalities do not generate indeterminacy, in one-sector growth model with exogenous labor supply. Moreover, we know from Alonso-Carrera, Caballé and Raurich [1] that consumption externalities are a source of indeterminacy in one-sector growth model with endogenous labor supply. So, in this paper, we will show, in the two-sector growth model that endogenous labor supply is not necessary to have equilibrium indeterminacy with consumption externalities. Moreover, we will see the indeterminacy results are only linked to consumption externality in preferences (i.e. keeping up with the joneses). The major difference between the results of Alonso-Carrera, Caballé and Raurich [1] and ours results is that we do not make any assumption on the weight of consumption externality (i.e. on the sensibility of the representative agent according to the consumption standard) whereas they suppose the consumption externality plays a neglicted role compared to the consumption. Consequently, we choose to interpret the relative consumption of the representative agent (i.e. the ratio between his own consumption and the consumption standard) as his social status. Hence, we do not make any assumption on the weight of consumption externality so we can suppose that the social status may have the same kind of importance than the consumption for the representative agent. We also let the representative agent have a behaviour which maintains his social status over time. We will see, such situations create a new mechanism for sunspot fluctuations. First, to obtain general results we do not specify the utility function. Then, we take a Cobb-Douglas utility function to illustrate the relationship between social statuss and own consumption.

Consequently, keeping up with the Joneses in the optimal growth continous-time model with two-sector leads to a new mechanism of sunspot fluctuations even if the consumption good is labor intensive. The labor intensive configuration for consumption good in the continous-time two setor model is quite new in the litterature on endogenous cycles but it seems more satisfying according the empiral results.

The rest of the paper is organized as follows. Section 2 describes the economy. Section 3 characterizes the competitive equilibrium. Section 4 gives formal conditions to obtains local indeterminacy. Section 5 analyzes the mechanism that leads to equilibrium indeterminacy.

Section 6 gives an example of utility function that allows the existence of indeterminacy and illustrates our main result through a standard parametrization of the model. Finally, Section 7 concludes. All the proofs are collected in the appendix (section 8).

2 The economy

We consider an infinite horizon, continous-time, two-sector model with Cobb-Douglas technologies, inelastic labor supply and consumption externalities through "keeping up with the Joneses" preferences. The economy consists of competitive firms and a representative household.

2.1 Firms

We assume that consumption good y_0 and capital good y_1 are produced by capital x_{1j} and labor x_{0j}, $j = 0, 1$, through a Cobb-Douglas technology. Hence, the "private production function" used by the representative firm in each industry is given by :

$$y_j = F_j\left(x_{0j}, x_{1j}\right) = x_{0j}^{\beta_j} x_{1j}^{1-\beta_j} \quad \text{for } j = 0, 1 \tag{1}$$

with $\beta_j \in [0, 1]$.

Therefore the returns to scale are constant, in each sector $j = 0, 1$.

We normalize the price of consumption good to one i.e. $p_0 = 1$. The representative firm in each sector $j = 0, 1$ maximizes its profit π_j given the output price p_j of the good produced in sector j, the wage rate w_0 and the rental rate of capital w_1, subject to production function 1. The first order conditions give :

$$\begin{array}{r} a_{0j}(w_1, p_j) = \frac{\beta_j p_j}{w_i} = \frac{x_{0j}}{y_j} \\ a_{1j}(w_1, p_j) = \frac{(1-\beta_j) p_j}{w_i} = \frac{x_{1j}}{y_j} \end{array} \tag{2}$$

for the factor i in the sector j. We call a_{ij} the input coefficients. The factor-price frontier, which gives a relationship between input prices and output prices is expressed with these input coeffcients.

Lemma 1 *: Denote* $p = (1, p_1)'$, $x = (1, x_1)$, $y = (y_0, y_1)$, $w = (w_0, w_1)'$ *and* $A(w, p) = [a_{ij}(w_i, p_i)]$. *Then :*

$$p = A'(w, p)w$$
$$x = A(w, p)y$$

Note that at the equilibrium the wage rate and the rental rate are functions of the output price only i.e. $w_0 = w_0(p_1)$ and $w_1 = w_1(p_1)$ whereas outputs are functions of the capital stock and the output price i.e. $y_j = y_j(x_1, p_1)$ for $j = 0, 1$.

2.2 Household

We assume that the population is constant and normalized to one. The representative agent derives his utility from consumption according to function $U(c(t), z(t))$,where $c(t)$ is the individual consumption, $z(t)$ the consumption externality given by the consumption average of the economy which can be interpreted as the consumption standard. The function U satisfies the standard hypotheses on the behavior of the consumer with respect to the consumption : the marginal utility in consumption is positive and decreasing. Moreover, the introduction of consumption standard implies that consumption spillovers affect the household's utility. Indeed, we assume that preferences correspond to the "keeping up with the Joneses" formulation such that the marginal utility of consumption rises with the consumption standard. Hence the following assumption holds :

Assumption 1 *:* $U(., z(t))$ *is increasing and concave* $\forall z(t) \in \mathbb{R}^+$*, the first partial derivatives satisfy* $U_z(c(t), z(t)) > 0$ *and* $U_c(c(t), z(t)) > 0$ *and the second partial derivatives satisfy* $U_{cz}(c(t), z(t)) > 0$ *and* $U_{cc}(c(t), z(t)) < 0$

We introduce the following elasticities :

$$\epsilon_{cc} = -\frac{U_c(c(t), z(t))}{c(t)U_{cc}(c(t), z(t))} > 0 \quad (3)$$

$$\epsilon_{cz} = \frac{U_c(c(t), z(t))}{c(t)U_{cz}(c(t), z(t))} > 0 \quad (4)$$

where ϵ_{cc} is the elasticity of intertemporal susbstitution in consumption and ϵ_{cz} is the absolute value of the elasticity of substitution between consumption and consumption standard.

The enhanced value of the consumption standard by the agent is positively correlate to ϵ_{cz}. We introduce the ratio $\frac{c(t)}{z(t)}$ as the relative consumption of the agent (i.e. the ratio between his own consumption and the consumption standard of the economy). Note that this ratio can be interpreted as his social status.

The elasticity ϵ_{cz} can be interpreted as a measure of the importance given by the agent to his social status. In most of the models using "keeping up with the Joneses" formulations, it is common to assume that $\epsilon_{cz} < \epsilon_{cc}$ i.e. $U_{cc} + U_{cz} < 0$ in order to have a consumption externality that has smaller effects on the preference of the representative agent compared to his own consumption. Consequently we do not make any assumption on the weight of consumption externality since we suppose that the social status may have the same kind of importance than the consumption for the representative agent [1]. Therefore, we can have $\epsilon_{cc} < \epsilon_{cz}$ i.e. $U_{cc} + U_{cz} > 0$ that is, we consider that the agents may give more weight or more importance to their social status rather than to their own present consumption.

The objective of the representative agent is to solve the following intertemporal optimization problem by taking $z(t)$ as given :

$$\begin{array}{ll} \max\limits_{y_1(t), x_1(t)} & \int_0^\infty e^{-\delta t} U\left(c(t), z(t)\right) dt \\ \text{s.c.} & \dot{x}(t) = y_1(t) - g x_1(t) \\ & x_1(0) = x_1 \text{ given} \end{array} \tag{5}$$

Where $\delta > 0$ is the subjective discount rate and $g \in (0, 1)$ the depreciation rate of capital. The production frontier is defined as :

$$c = T(x_1, y_1) = x_{00}^{\beta_0} x_{01}^{1-\beta_0}$$

1. We note that there is no problem about the concavity of the utility function when we suppose that $\epsilon_{cc} < \epsilon_{cz}$ as z(t) is take as given by the representative agent.

From the enveloppe theorem we easily get $w_1 = T_1(x_1, y_1)$ and $p_1 = -T_2(x_1, y_1)$ [2].

The Hamiltonian in current value of (5) is :

$$H = U(T(x_1, y_1), z(t)) + q_1(t)(y_1(t) - gx_1(t)) \tag{6}$$

The first order conditions are :

$$\begin{aligned} p_1(t)U_c(c(t)) &= q_1(t) && (7) \\ q_1(t)(\delta + g) - w_1 U_c(c(t)) &= \dot{q}_1(t) && (8) \\ y_1 - gx_1 &= \dot{x}_1 && (9) \end{aligned}$$

$$\lim_{t \to +\infty} x_1(t) U_c(c(t)) p_1(t) e^{-\rho . t} = 0 \tag{10}$$

Where q_1 is the co-state variable which corresponds to the utility price of capital in current value.

3 The competitive equilibrium

Let us denote :

$$\alpha = \frac{1}{\epsilon_{cc}} - \frac{1}{\epsilon_{cz}} \tag{11}$$

If $\epsilon_{cc} < (>)\ \epsilon_{cz}$ then we have $\alpha > (<) 0$.

To obtain the dynamic equations characterizing the symetric equilibrium i.e $z = c$, we combine (7) and (8) and since $c = c(x_1, p_1)$, after a total differentiation of (7), we have the two equations of motion which describe the dynamic of equilibrium paths :

$$\begin{aligned} \dot{x}_1 &= y_1(x_1, p_1) - gx_1 && (12) \\ \dot{p}_1 &= \frac{1}{E(x_1, p_1)} \left[p_1(\delta + g) - w_1(p_1) + \alpha \frac{p_1}{c} \frac{\partial c}{\partial x_1} (y_1(x_1, p_1) - gx_1) \right] \end{aligned}$$

with

$$E(x_1, p_1) = 1 - \alpha \frac{p_1}{c} \frac{\partial c}{\partial p_1} \tag{13}$$

2. Note that $y_0 = c(x_1, p_1) = T(x_1, y_1(x_1, p_1))$.

3.1 Steady state

Any solution $\{x_1(t), p_1(t)\}_{t\geqslant 0}$ of the system (12) satisfying the transversality condition (10) will be called equilibrium path.

Proposition 1 *There exists a unique steady state* $(x_1^*, p_1^*) > 0$ *solution of :*

$$\begin{aligned}\dot{x}_1 &= 0 \Longleftrightarrow y_1\left(x_1, p_1\right) = gx_1 \\ \dot{p}_1 &= 0 \Longleftrightarrow w_1(p_1) = p_1\left(\delta + g\right)\end{aligned}$$

See appendix 8.1 for details.

3.2 Caracteristic polynomial

In order to study the indeterminacy properties of equilibrium, we linearize the system (12) around (x_1^*, p_1^*) which gives the following Jacobian :

$$J = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} - g & \frac{\partial y_1}{\partial p_1} \\ \frac{\alpha}{E}\frac{p_1}{c}\frac{\partial c}{\partial x_1}\left(\frac{\partial y_1}{\partial x_1} - g\right) & \frac{1}{E}\left[\delta + g - \frac{\partial w_1}{\partial p_1} + \alpha\frac{p_1}{c}\frac{\partial c}{\partial x_1}\frac{\partial y_1}{\partial p_1}\right] \end{pmatrix} \quad (14)$$

Any solution from (12) that converges to the steady state (x_1^*, p_1^*) satisfies the transversality condition and is an equilibrium. Therefore, given initial capital stock $x_1(0)$ if there is more than one initial price $p_1(0)$ in the stable manifold of (x_1^*, p_1^*), the equilibrium path coming from $x_1(0)$ will not be unique. In particular, if the Jacobian matrix J (14) has two eigenvalues with negative real part (the locally stable manifold of the steady state (x_1^*, p_1^*) is two dimensional), there will be a continuum of converging paths and thus a continuum of equilibria : (x_1^*, p_1^*) is said to be locally indeterminate.

The dynamics of the model around the steady state can be fully derived from the eigenvalues of Jacobian matrix (14) or from the roots λ_1 and λ_2 of the caracteristic polynomial :

$$P\left(\lambda\right) = \lambda^2 - T\lambda + D \quad (15)$$

where T and D are respectively the trace and the determinant of the Jacobian matrix (14).

The trace and the determinant T and D are given by :

$$\begin{aligned} T = \lambda_1 + \lambda_2 &= \tfrac{1}{E}\Big\{ \tfrac{\partial y_1}{\partial x_1} + \delta - \tfrac{\partial w_1}{\partial p_1} + \alpha \tfrac{p_1}{c} \\ &\quad \left(\tfrac{\partial c}{\partial x_1}\tfrac{\partial y_1}{\partial p_1} - \tfrac{\partial c}{\partial p_1}\left(\tfrac{\partial y_1}{\partial x_1} - g\right)\right)\Big\} \qquad (16) \\ D = \lambda_1 \lambda_2 &= \tfrac{1}{E}\left(\tfrac{\partial y_1}{\partial x_1} - g\right)\left(\delta + g - \tfrac{\partial w_1}{\partial p_1}\right) \qquad (17) \end{aligned}$$

We know that the steady state is locally indeterminate if and only if $T < 0$ et $D > 0$. Therefore, we have to study the sign of T and D in function of b and α.

4 Existence of local indeterminacy

Our main objective is to study the impact of consumption externality measured by the elasticity ϵ_{cz} on the local determinacy properties of the long-run equilibrium.

Solving the system (16-17) with respect to α gives a linear relationship between $T(\alpha)$ and $D(\alpha)$: when α varies, $T(\alpha)$ and $D(\alpha)$ move along the line called in what follows Δ_α(see appendix 8.3 for the proof), which is defined by [3] :

$$D = S_\alpha T + M_\alpha$$

with

$$S_\alpha = \frac{\frac{\partial c}{\partial p_1}\left(\frac{\partial y_1}{\partial x_1} - g\right)\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right)}{\frac{\partial c}{\partial x_1}\frac{\partial y_1}{\partial p_1} + \frac{\partial c}{\partial p_1}\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right)} \qquad (18)$$

$$M_\alpha = \frac{\left(\frac{\partial y_1}{\partial x_1} - g\right)\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right)\left(\frac{\partial c}{\partial x_1}\frac{\partial y_1}{\partial p_1} - \frac{\partial c}{\partial p_1}\left(\frac{\partial y_1}{\partial x_1} - g\right)\right)}{\frac{\partial c}{\partial x_1}\frac{\partial y_1}{\partial p_1} + \frac{\partial c}{\partial p_1}\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right)} \qquad (19)$$

Note that S_α and M_α depend only upon technological parameters.

In their previous paper, Garnier, Venditti and Nishimura [11] have already studied the case where the utility function is non linear, under the

3. Note that (x_1^*, p_1^*) does not depend on α and remains the same along the line Δ_α.

presence of sector-specific externalities such there is capital intensity reversal between the private and the social level, they have proved the existence of sunspot fluctuations is obtained if and only if the elasticity of intertemporal substitution in consumption is large enough and the elasticity of labor supply is low enough (even equal to zero). Therefore, we will see that without externalities (i.e. constant returns to scale), it is possible to have a room of local indeterminacy led by keeping up with the Joneses. The case where $\alpha > 0$ cannot provide local indeterminacy, indeed, in the two-sector model without consumption externalities, Garnier, Venditti and Nishimura [11] have shown that it must have sector specific externalities to have local indetermincy even if the utility function is non linear (which corresponds to the case where $\alpha = 1/\epsilon_{cc}$). Therefore, we can deduct that whereas the level of consumption externality is low (i.e.$\epsilon_{cc} < \epsilon_{cz}$ and $\alpha > 0$), the presence of consumption externality cannot change the results obtained by Garnier, Venditti and Nishimura [11] . Consequently, we focus only on the case where the level of consumption externality is sufficiently large to have $\epsilon_{cc} > \epsilon_{cz}$ and thus $\alpha < 0$.

We use the geometrical method of Grandmont, Pintus and De Vilder [13] in order to study the variations of $T(\alpha)$ and $D(\alpha)$ in the (T, D) plane, when α varies continuously on $]-\infty, 0[$.

We make the following assumption :

Definition 1 *: The consumption good is said to be capital (labor) intensive if and only if :*

$$a_{00}a_{11} - a_{01}a_{10} < (>) 0$$

At the steady state, it's possible to give this last condition only with the technological parameters β_{ij} :

Proposition 2 *: Let $b \equiv \beta_0 - \beta_1$. At the steady state we have : $a_{00}a_{11} - a_{01}a_{10} < (>) 0 \Leftrightarrow b < (>) 0$*

So, when α gets from $-\infty$ to 0, the pair $(T(\alpha), D(\alpha))$ moves along the half line Δ_α [4] characterized by a starting point $(T(\infty), D(\infty))$:

$$T(\infty) = -\frac{\frac{\partial c}{\partial x_1}\frac{\partial y_1}{\partial p_1}}{\frac{\partial c}{\partial p_1}} + \frac{\partial y_1}{\partial x_1} - g \quad (20)$$

$$D(\infty) = 0 \quad (21)$$

and a end point $(T(0), D(0))$ such that :

$$D(0) = \left(\frac{\partial y_1}{\partial x_1} - g\right)\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right) \quad (22)$$

$$T(0) = \delta \quad (23)$$

Consequently, as the ending point is characterized by $D(0) < 0$ (since $\frac{\partial y_1}{\partial x_1} = \frac{\partial w_1}{\partial p_1}$) and $T(0) = \delta$ then indeterminacy is ruled out for $\alpha = 0$ whereas the starting point is located on the abscissa area since $D(\infty) = 0$.

If we can verify both $T(\infty) < 0$ and D is an increasing function of α then the half line Δ_α gets by the indeterminacy area ($T < 0$ and $D > 0$). We have to give conditions to have $T(\infty) < 0$ and $\frac{dD}{d\alpha} = \frac{p_1^*}{c^* E^2}\frac{\partial c}{\partial p_1} D(0) > 0$. We can see immediately that $\frac{dD}{d\alpha}$ depends only on the sign of the derivative $\frac{\partial c}{\partial p_1}$ since $D(0) < 0$ and then we must verify $\frac{\partial c}{\partial p_1} < 0$. In this case, the pair $(T(\alpha), D(\alpha))$ enters in the indeterminacy area when α gets from $-\infty$ to a critical value $\bar{\alpha}$ then it gets out of the indeterminacy area and finishes on the point $(\delta, D(0))$. Otherwise, $T(\infty) > 0$ implies $\frac{dD}{d\alpha} < 0$ and the pair $(T(\alpha), D(\alpha))$ moves in the wrong way : local indeterminacy is ruled out (see appendix 8.5 and 8.4).

Proposition 3 *: Suppose that* $\beta_1 > \underline{\beta_1} \equiv \frac{\delta}{2\delta+g}$ *, then there exists* $\beta^*_{\beta_1} \equiv -\frac{\frac{g}{\delta+g}\beta_1(1-\beta_1)}{(1-2\beta_1-g\frac{(1-\beta_1)}{\delta+g})}$ [5] > 0 *such that* $\forall \beta_0 > \beta^*_{\beta_1}$ *we have* $\frac{dD}{d\alpha} > 0$ *and* $T(\infty) < 0$.

(Proof : see appendix 8.5 and 8.4)

4. When $\alpha > 0$, the pair $(T(\alpha), D(\alpha))$ moves along the another part of the half line Δ_α : see Garnier, Venditti and Nishimura [12].

5. Where $\beta^*_{\beta_1}$ is a critical value that depends on the value that we have choosen for the labor share in the investment sector β_1.

Assumption 2 *: The labor share of the investment sector verifies :* $\beta_1 > \underline{\beta_1} \equiv \frac{\delta}{2\delta+g}$.

Finally, when 3 is held and since Δ_α is a half-line and $D(\alpha)$ is an increasing function of α and $T(\infty) < 0$, the pair $(T(\alpha), D(\alpha))$ gets by the indeterminacy area when α increases from $-\infty$ to 0 and finishs on the point $(T(0), D(0))$ such that $T(\infty) < 0$ and $D(\infty) = 0^+$ (as the pair $(T(\alpha), D(\alpha))$ comes from the half-plan where the values of $D(\alpha)$ are positive). More precisely, there exists $\bar{\alpha} \in]-\infty, 0[$ such that the steady state is locally indeterminate $\forall\alpha \in]-\infty, \bar{\alpha}[$.

Hence, we give the following lemma :

Lemma 2 *: Under assumptions 1 and 2 : suppose that* $\beta_1 > \frac{\delta}{2\delta+g}$ *,* $\forall\beta_0 > \beta^*_{\beta_1}$

$\exists\overline{\alpha} \equiv \frac{1}{\frac{p_1}{c}\frac{dc}{dp_1}} \in]-\infty, 0[$ *such that* $\forall\alpha < \overline{\alpha}$ *the steady state* (x_1^*, p_1^*) *is locally indeterminate and* $\forall\alpha > \overline{\alpha}$ *the steady state* (x_1^*, p_1^*) *is saddle point.*

This lemma gives a condition set which gives endogenous fluctuations without any restriction on the factor intensity of the consumption good. Indeed, there is just a critical value on the labor share in the consumption sector $\beta^*_{\beta_1}$(linked on the labor share of the investment sector β_1) and a critical value of the labor share in the investment sector through assumption 2 (linked on the discount time and depreciation rate) ; it is sufficient to choose $\beta_0 > \beta^*_{\beta_1}$ to obtain local indeterminacy for any quite small value of α (i.e. α must be smaller than $\overline{\alpha}$). Therefore, endogenous fluctuations don't depend on the factor intensity of the consumption sector. This result is in sharp constrast with the ones of Benhabib, Nishimura and Venditti [8] or Nishimura, Takahashi and Venditti [18].

5 Endogenous fluctuations mechanism

This result means that consumption externality like "keeping up with the Joneses" gives rise to new mechanism of indeterminacy which is not linked to the one coming from technological externalities.

We can provide an heuristic explanation to understand how the "keeping up with the Joneses" leads to local indeterminacy. We know that α has to be negative to lead to the equilibrium indeterminacy. When $\alpha < 0$, the agent favours his social status $\frac{c(t)}{z(t)}$ more than his own consumption $c(t)$ over time.

Now, starting from an arbitrary equilibrium, consider that the agent expects another one with a higher rate of investment and higher level of capital stock coming from an instantaneous increase in relative price of investment good p_1. The only way that this other equilibrium path becomes a new equilibrium path is to find a mechanism which reverses the price toward the equilibrium and offsets this initial increase. Since the labor supply is exogenous, if one of the both goods (consumption or investment) increases (decreases) then the other one decreases (increases). Suppose, for example, that the investment good is capital intensive ($b > 0$), then a higher capital stock implies an increase (more than proportional) of output of the investment good and a decrease (more than proportional) of output of the consumption good and thus in the own consumption of the representative agent and in his social status. But the representative agent, who gives more value to his social status than his consumption, wants to keep his social status over time and thus has to increase his future consumption. The level of this rise in the consumption depends on the level of the externality measured by ϵ_{cz}. Indeed, there are two opposite effects playing through the decreasing marginal utility in consumption (i.e. $U_{cc} < 0$) and through the positive effect of externality on this marginal utility (i.e. $U_{cz} > 0$). Since, we have assumed that $U_{cc} + U_{cz} > 0$, the overall effect of this decrease in the present consumption and in the social status leads to a decrease in the marginal utility. Therefore, the representative agent has to increase more than proportional his future consumption to conserve his level of utility. Consequently, the present decrease in the consumption of the representative leads to a large increase of his future level of consumption. To support this future level of consumption the output in this sector will have to increase and as the labor is exogenous, that will lead to a large decrease of the output in the investment sector what will reverse its price p_1 toward the equilibrium and offsets its initial rise [6]. But, to ensure that this increase of level of consumption has a sufficient decreasing impact on the output of the investment sector, the labor share in each sector has to be greater than a critical value which depends only on the depreciation rate g and discount time δ for the investment sector (i.e assumption 2) but depends also on the labor share $\beta_1$of the investment sector for the consumption sector (i.e. $\beta_0 > \beta^*_{\beta_1}$).

6 Examples

Let us consider Cobb-Douglas formulation between the social status $\frac{c(t)}{z(t)}$ and the consumption $c(t)$ such as :

$$U(c(t), z(t)) = \frac{1}{1-\sigma}\left[c(t)^{1+\gamma}\left(\frac{c(t)}{z(t)}\right)^{-\gamma}\right]^{1-\sigma}$$

where $\beta > 0$ represents the weight of the consumption externality or the sensibility of the representative agent according to the standard of consumption $z(t)$, and $\sigma > 0$ represents the individual risk aversion. This function satisfies assumption 1. We can derive the following elasticities :

$$\epsilon^*_{cc} = \frac{1}{\sigma}$$
$$\epsilon^*_{cz} = -\frac{1}{\gamma(1-\sigma)}$$

Therefore :

$$\alpha = \sigma(1+\gamma) - \gamma$$

Note that for any $\gamma > 0$, if $\sigma = 0$ (i.e. the utility is linear with respect to the consumption) then $\alpha < 0$ and if $\sigma = 1$ then $\alpha = 1$ Therefore $\forall\gamma > 0$, $\exists\tilde{\sigma} \in [0, 1[$ such that $\forall\sigma \in [0, \tilde{\sigma}[$ we have $\alpha < 0$.

We want to illustrate the lemma 2) for both capital and labor intensive consumption good

6. We note that there is a symetric mechanism, but in the opposite way, when the investment good is labor intensive.

6.1 Labor or capital intensive consumption good

We use a parametrization according empirical results on the labor share in european countries :

$$\beta_1 = 0.62$$
$$g = 0.05$$
$$\delta = 0.01$$

This configuration gives$\underline{\beta_1} = 0.142$ and $\beta^*_{\beta_1} = 0.35$, that means that we can take any value greater than 0.142 for the labor share of the investment sector and greater than 0.35 for the consumption sector to obtain endogenous fluctuations. Moreover we can choose this value in order to have labor intensive or capital intensive consumption good. For example, if we set $\beta_0 = 0.6$ then we have $b = -0.02$ and the consumption good is capital intensive, moreover, we obtain the following critical value : $\overline{\alpha} = -0.05$, that is we can set, for example, $\sigma = 0.2$ and $\gamma = 0.32$ (i.e. all value of σ and γ such that $\sigma(1+\gamma) - \gamma < -0.05$. Otherwise, if we set $\beta_0 = 0.65$ then we have $b = 0.03$ and from now, the consumption good is labor intensive, moreover, we obtain a new critical value for the inverse of the elasticity of intertemporal substitution in consumption : $\overline{\alpha} = -0.01$, for example $\sigma = 0.2$ and $\gamma = 0.27$.

7 Concluding comments

In this paper we have shown that a new mechanism can bring endogenous fluctuations. It comes only from preferences and the behavior of the representative agent toward his social status. Since Benhabib and Nishimura [7], endogenous fluctuations were the result of market imperfections coming from technological externalities leading to a capital intensity reversal between the private and social level. Our new mechanism is totaly independent of that of Benhabib and Nishimura. Moreover, the consumption good may be capital or labor intensive by opposite to the model with market imperfection where the endogenous fluctuation need a capital intensive consumption good. This mechanism depends on the behavior of the representative agent : he has to give more value to his social status than his own present consumption.

The most important results is that we have a mechanism of endogenous fluctuations that not implies capital intensities reversal or capital intensive consumption goo, but only the assumption that the representative agent gives some value to his social status. The direct consequence is that, for the first time in this type of model, we have endogenous fluctuations under constant returns to scale, that is without any type of technological externalityor market imperfections, moreover with labor intensive sectors.

8 Appendix

8.1 Proof of existence of (x_1^*, p_1^*)

The maximization of profit gives the following first order conditions :

$$w_i = p_j \beta_{ij} \frac{y_j}{x_{ij}} \quad \text{for} \quad i,j = 0,1 \tag{24}$$

The steady state is characterized by : $y_1 = gx_1$ and $w_1 = (\delta + g)\, p_1$, so that :

$$x_{11} = \frac{1-\beta_1}{\delta + g} g x_1 \tag{25}$$

Moreover we have :

$$x_{01} = g x_1 \left(\frac{1-\beta_1}{\delta + g} \right)^{-\frac{1-\beta_1}{\beta_1}} \tag{26}$$

The stock equations : $x_1 = x_{10} + x_{11}$ et $1 = x_{01} + x_{00}$ allow to give :

$$x_{10} = x_1 \left(1 - g \frac{1-\beta_1}{\delta + g} \right) \tag{27}$$

$$x_{00} = 1 - g x_1 \left(\frac{\beta_{11}}{\delta + g} \right)^{-\frac{1-\beta_1}{\beta_1}} \tag{28}$$

From (24) we have :

$$\frac{x_{00} x_{11}}{x_{01} x_{10}} = \frac{(1-\beta_1)\beta_0}{(1-\beta_0)\beta_1} \tag{29}$$

From (25), (26), (27), (28), (29), we have :

$$x_1^* = \frac{\left(\frac{1-\beta_1}{\delta+g}\right)^{\frac{1}{\beta_1}}}{\frac{1-\beta_1}{\delta+g} g \left(1 - \frac{(1-\beta_1)\beta_0}{(1-\beta_0)\beta_{10}}\right) + \frac{(1-\beta_1)\beta_0}{(1-\beta_0)\beta_1}} > 0 \tag{30}$$

For $i = 1$and $j = 0$, (24), (27) and (30) give :

$$w_1^* = (1-\beta_0) \left(\frac{(1-\beta_1)\beta_0}{(1-\beta_0)\beta_1}\right)^{\beta_{00}} \left(\frac{1-\beta_1}{\delta+g}\right)^{-\frac{\beta_0}{\beta_1}} \tag{31}$$

We derive :

$$p_1^* = \frac{1-\beta_0}{\delta+g} \left(\frac{(1-\beta_1)\beta_0}{(1-\beta_0)\beta_1}\right)^{\beta_{00}} \left(\frac{1-\beta_1}{\delta+g}\right)^{-\frac{\beta_0}{\beta_1}} > 0 \tag{32}$$

8.2 Computation of derivatives used in $T(\alpha)$ and $D(\alpha)$

In order to compute (16) and (17) we need the following partial derivatives : $\frac{\partial y_1}{\partial x_1}$, $\frac{\partial c}{\partial x_1}$, $\frac{\partial w_1}{\partial p_1}$,$\frac{\partial y_1}{\partial p_1}$, $\frac{\partial c}{\partial p_1}$.

To compute $\frac{\partial y_1}{\partial p_1}$and $\frac{\partial c}{\partial p_1}$ we begin by the total differentiation of the quantity equations given by :

$$\begin{aligned} a_{00}y_0 + a_{01}y_1 &= 1 \\ a_{10}y_0 + a_{11}y_1 &= x_1 \end{aligned}$$

The total differentiation gives :

$$a_{00}dy_0 + a_{01}dy_1 + \frac{\partial a_{00}}{\partial w_0} y_0 dw_0 + y_1 \left(\frac{\partial a_{01}}{\partial w_0} dw_0 + \frac{\partial a_{01}}{\partial p_1} dp_1\right) = 0 \tag{33}$$

$$a_{10}dy_0 + a_{11}dy_1 + \frac{\partial a_{10}}{\partial w_1} y_0 dw_1 + y_1 \left(\frac{\partial a_{11}}{\partial w_1} dw_1 + \frac{\partial a_{11}}{\partial p_1} dp_1\right) = dx_1 \tag{34}$$

After, we need $\frac{\partial c}{\partial x_1}$ and $\frac{\partial y_1}{\partial x_1}$ with $c = y_0$ and $dw_0 = dw_1 = dp_1 = 0$. Then, we have :

$$\frac{\partial y_1}{\partial x_1} = \frac{a_{00}}{a_{11}a_{00} - a_{10}a_{01}} \tag{35}$$

$$\frac{\partial c}{\partial x_1} = -\frac{a_{01}}{a_{11}a_{00} - a_{10}a_{01}} \tag{36}$$

These derivatives correspond to the Rybczynsky effect.

Now, compute $\frac{\partial y_1}{\partial p_1}$ and $\frac{\partial c}{\partial p_1}$. With the price equations given by :

$$\begin{aligned} a_{00}w_0 + a_{10}w_1 &= 1 \\ a_{01}w_0 + a_{11}w_1 &= p_1 \end{aligned}$$

So :

$$\frac{\partial w_1}{\partial p_1} = \frac{\partial y_1}{\partial x_1} \tag{37}$$

$$\frac{\partial w_0}{\partial p_1} = -\frac{a_{10}}{a_{11}a_{00} - a_{10}a_{01}} \tag{38}$$

On the other hand we derive from

$$\begin{aligned} a_{0j}(w_1, p_j) = \tfrac{\beta_j p_j}{w_i} = \tfrac{x_{0j}}{y_j} \\ a_{1j}(w_1, p_j) = \tfrac{(1-\beta_j)p_j}{w_i} = \tfrac{x_{1j}}{y_j} \end{aligned}$$

that :

$$\frac{\partial a_{ij}}{\partial w_i} = -\frac{a_{ij}}{w_i} \tag{39}$$

$$\frac{\partial a_{ij}}{\partial p_j} = \frac{a_{ij}}{p_j} \tag{40}$$

We substitute (37), (38), (39) et (40) in (33) and (34). Hence, the resolution of the system (33) and (34), with $dx_1 = 0$ give :

$$a_{00}dy_0 + a_{01}dy_1 + dp_1 \left(\frac{a_{01}}{p_1} y_1 - \left(p_1 - \frac{\hat{a}_{11}}{\hat{a}_{10}} \right)^{-1} \right) = 0 \tag{41}$$

$$a_{10}dy_0 + a_{11}dy_1 + dp_1 \left(\frac{a_{11}}{p_1} y_1 - x_1 \left(p_1 - \frac{\hat{a}_{01}}{\hat{a}_{00}} \right)^{-1} \right) = 0 \tag{42}$$

Using (41) and (42) we have thus :

$$\frac{\partial y_1}{\partial p_1} = \frac{a_{00}}{a_{11}a_{00} - a_{10}a_{10}} \frac{1}{p_1} \left(\frac{\beta_0}{\beta_0 - \beta_1} x_1 - \frac{a_{10}}{a_{00}} \frac{1 - \beta_0}{\beta_0 - \beta_1} \right) - \frac{g x_1}{p_1} \tag{43}$$

$$\frac{\partial c}{\partial p_1} = -\frac{a_{01}}{a_{11}a_{00} - a_{10}a_{10}} \frac{1}{p_1} \left(\frac{\beta_0}{\beta_0 - \beta_1} x_1 - \frac{a_{11}}{a_{01}} \frac{1 - \beta_0}{\beta_0 - \beta_1} \right) \tag{44}$$

8.3 Computation of Δ_α

Consider the expressions $T(\alpha)$ (16), $D(\alpha)$ (17) and $E = 1 - \alpha.\frac{p_1}{c}.\frac{\partial c}{\partial p_1}$
With (16) and (17) we can extract α :

$$\alpha = \frac{D(\alpha) - \left(\frac{\partial y_1}{\partial x_1} - g\right)\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right)}{\frac{p_1}{c}\frac{\partial c}{\partial p_1} D(\alpha)} = \frac{T(\alpha) - \left(\frac{\partial y_1}{\partial x_1} + \delta - \frac{\partial w_1}{\partial p_1}\right)}{\frac{p_1}{c}\frac{\partial c}{\partial p_1} T(\alpha) + \frac{p_1}{c}\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right)} \tag{45}$$

Therefore :

$$D = \frac{\frac{\partial c}{\partial x_1}\left(\frac{\partial y_1}{\partial x_1} - g\right)\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right)}{\frac{\partial c}{\partial x_1}\frac{\partial y_1}{\partial p_1} + \frac{\partial c}{\partial p_1}\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right)} T + \frac{\left(\frac{\partial y_1}{\partial x_1} - g\right)\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right)\left(\frac{\partial c}{\partial x_1}\frac{\partial y_1}{\partial p_1} - \frac{\partial c}{\partial p_1}\left(\frac{\partial y_1}{\partial x_1} - g\right)\right)}{\frac{\partial c}{\partial x_1}\frac{\partial y_1}{\partial p_1} + \frac{\partial c}{\partial p_1}\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right)} \tag{46}$$

where S_α is given by (18) and M_α is given by (19)

The computation of the derivative $\frac{dT}{d\alpha}$ and $\frac{dD}{d\alpha}$ give :

$$\frac{dT}{d\alpha} = \frac{p_1^*}{c^* E^2}\left[\frac{\partial c}{\partial x_1}\frac{\partial y_1}{\partial p_1} + \frac{\partial c}{\partial p_1}\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right)\right] \tag{47}$$

$$\frac{dD}{d\alpha} = \frac{p_1^*}{c^* E^2}\frac{\partial c}{\partial p_1}\left(\frac{\partial y_1}{\partial x_1} - g\right)\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right) \tag{48}$$

When technological parameters are fixed, only E depends on α in the expression of these derivatives. Hence, it is easy to show that the sign of $\frac{\partial T}{\partial \alpha}$ and $\frac{\partial D}{\partial \alpha}$ does not depend on α and remains constant $\forall\ \alpha$.

8.4 Proof of Proposition 3 $\frac{dD}{d\alpha} < 0$

We have to prove that $\frac{dD}{d\alpha} > 0$. Since we can rewrite 48 as $\frac{dD}{d\alpha} = \frac{p_1^*}{c^* E^2}\frac{\partial c}{\partial p_1} D(0)$ and since $D(0)$ is always negative we have to give conditions to have $\frac{dc}{dp_1} < 0$. From 40, we can rewrite $\frac{\partial c}{\partial p_1}$ with the parameters of the model :

$$\frac{\partial c}{\partial p_1} = x_1(\delta + g)\left\{\frac{\beta_1(1 - 2\beta_0)}{b^2} + \left(1 - g\frac{1 - \beta_1}{\delta + g}\right)\frac{1}{b}\right\} \tag{49}$$

When $b < (>)0$ we have $\beta_0 < (>)\beta_1$ and since $1 - g\frac{1-\beta_1}{\delta+g} > 0$ (since $\frac{g}{\delta+g} < 1$ and $1 - \beta_1 < 1$), consequently, we have several cases where $\frac{\partial c}{\partial p_1} < 0$.

Let $\beta^*_{\beta_1} \equiv -\frac{\frac{g}{\delta+g}\beta_1(1-\beta_1)}{(1-2\beta_1-g\frac{(1-\beta_1)}{\delta+g})}$ a critical value which depends on the labor share in the investment sector such that we have $\frac{\partial c}{\partial p_1} > 0$, $\forall \beta_0 > \beta^*_{\beta_1}$, where the critical value $\beta^*_{\beta_1}$ depends on the labor share in the investment sector β_1, that we have setted but not on the factor intensity configuration of sectors. Moreover, we have to ensure that this critical value is positive, that is garanted by $\beta_1 > \frac{\delta}{2\delta+g}$. Consequently : when $\beta_1 > \frac{\delta}{2\delta+g}$ is checked, then $\forall \beta_0 > \beta^*_{\beta_1}$ we have $\frac{\partial c}{\partial p_1} < 0$ and thus $\frac{dD}{d\alpha} > 0$.

8.5 Proof of Proposition 2 $T(\infty) < 0$

$T(\infty)$ (eq.15) is linked on the sign of the derivative $\frac{\partial c}{\partial p_1}$, $\frac{\partial y_1}{\partial p_1}$ and b (as we know that $b < 0$ gives $\frac{\partial y_1}{\partial x1} < 0$ and $\frac{\partial c}{\partial x_1} > 0$). As we know conditions which give $\frac{\partial c}{\partial p_1}$ positive, we have to study the impact of these ones on the sign of $T(\infty)$.

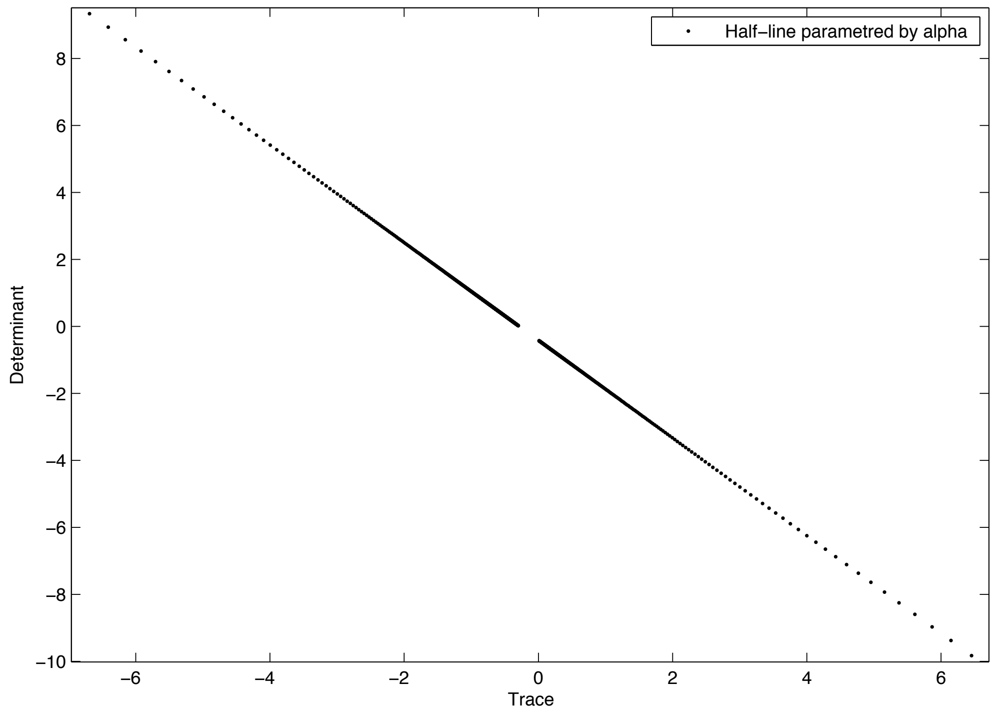
From 43, we can rewrite $\frac{\partial y_1}{\partial p_1}$ with the parameters of the model :

$$\frac{\partial y_1}{\partial p_1} = \frac{g x_1}{p_1}\left[\frac{1}{b^2}\frac{\delta+g}{g}\beta_0\left(2\beta_0-1\right)-1+\frac{1-\beta_0}{b}\right] \tag{50}$$

Now, from 20, 49 and 50 we can write $T(\infty)$ with the parameters of the model as :

$$T\left(\infty\right) = \frac{\frac{x_1}{b^2 w_0}\left[g\beta_1\left(1-\beta_0-b\right)+\beta_0\left(1-\beta_0\right)\left(\delta+g\beta_1\right)\right]}{\frac{\partial c}{\partial p_1}} - g \tag{51}$$

Since $\frac{\partial c}{\partial p_1}$ has to be negative we have to check that the sign of the numerator of $T(\infty)$ is positive. Moreover, to verify $\frac{\partial c}{\partial p_1} < 0$ we have to check that $\beta_0 > \beta^*_{\beta_1}$, which implies the numerator of $T(\infty)$ is positive and if $\frac{\partial c}{\partial p_1} < 0$ then $T(\infty)$ is always negative. Consequently, it explains why $T(\infty)$ must be negative and why $T(\infty) > 0$ rules out local indeterminacy.



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