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Thomas Baudin, David de La Croix, Paula Gobbi. DINKs, DEWKs & Co. Marriage, Fertility and Childlessness in the United States. 2012. hal-00993307

HAL Id: hal-00993307
https://hal.univ-lille.fr/hal-00993307v1

Preprint submitted on 20 May 2014

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Document de travail

■ [2012–21]

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Université Lille Nord de France
Pôle de Recherche et d'Enseignement Supérieur

Université Lille1 Sciences et Technologies
Université Lille 2 Droit et Santé
Lille 3 Université Sciences humaines et sociales Charles-de-Gaulle

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Thomas Baudin, David de la Croix and Paula Gobbi

Thomas Baudin
PRES Université Lille Nord de France, Université Lille 3, Laboratoire EQUIPPE EA 4018, Villeneuve d’Ascq, France.
thomas.baudin@univ-lille3.fr thomas.baudin@uclouvain.be

David de la Croix
IRES and CORE, Université catholique de Louvain
david.delacroix@uclouvain.be

Paula Gobbi
IRES, Université catholique de Louvain
paula.gobbi@uclouvain.be

DINKs, DEWKs & Co.
Marriage, Fertility and Childlessness in the United States

Thomas Baudin* David de la Croix† Paula Gobbi‡

November 7, 2012

Abstract

We develop a theory of marriage and fertility, distinguishing the choice to have children from the choice of the number of children. The deep parameters of the model are identified from the 1990 US Census. We measure voluntary and involuntary childlessness, and explain why (1) single women are more often childless than married women but, when mothers, their fertility are almost similar; (2) childlessness exhibits a U-shaped relationship with education for both single and married; (3) the relationship between marriage rates and education is hump-shaped. We show how family patterns have been shaped by the rise in education and wage inequality, and by the shrinking gender wage gap.

Keywords: Fertility, Childlessness, Marriage, Education, Structural Estimation.
JEL Classification Numbers: J11; O11; O40.

*EQUIPPE, Université de Lille 3 and IRES, Université catholique de Louvain. E-mail: thomas.baudin@uclouvain.be

†IRES and CORE, Université catholique de Louvain. E-mail: david.delacroix@uclouvain.be

‡IRES, Université catholique de Louvain. E-mail: paula.gobbi@uclouvain.be

[1] We acknowledge financial support from the Belgian French-speaking community (Grant ARC 009-14018 on "Sustainability"). Computational resources were provided by the supercomputing facilities of the Université catholique de Louvain (CISM/UCL). We would like to thank Matthias Doepke, Axel Gosseries, Marion Leturcq, Fabio Mariani, Víctor Ríos-Rull, Alice Schoonbroodt, Jean-Pierre Urbain, and other participants at seminars in Jerusalem, Barcelona, Paris I, Louvain-la-Neuve, Rostock, Mannheim, Northwestern University, University of Minnesota, Marseilles and Vigo for helpful comments and suggestions. All remained discrepancies and errors are ours, though we hope none of them will be such as to affect, in the least degree, the main arguments.

1 Introduction

American family patterns have changed over the twentieth century. Among those main transformations, the generalization of divorce has already been widely studied (see Becker (1991)). In this paper, we focus on the consequences of another fundamental change in the United States: the increasing discrepancy between marriage and motherhood. Nowadays, neither does marriage systematically imply parenthood nor does singleness mean childlessness. New types of families, such as the DEWKs (Dually Employed With Kids), the KOOPFs (Kids of One-Parent Families) or the DINKs (Double Income No Kids) have become more common.[1]

In this paper, we answer two questions. First, what are the incentives and constraints leading individuals to one type of family rather than another? In particular, when do married couples remain childless and when do single women become mothers? And second, how do economic changes affect the proportion of these different family types?

The model we propose to answer these questions will be challenged on its ability to account for three facts drawn from the U.S. Census Bureau data for the year 1990: (i) single women are much more likely to be childless, however, when they choose to become mothers, their fertility is almost the same as the fertility of married mothers,[2] (ii) there is a U-shaped relationship between childlessness and education both for single and for married women, and (iii) the relationship between marriage rates and education is hump-shaped. For the best of our knowledge, the first two facts have not been documented for both single and married women before and nobody has provided a theory accounting for the three facts altogether. All these facts are discussed in more detail in Section 2.

Childlessness can either be a life choice for the "child-free", or a heavy burden for those trapped in the impossibility of experiencing parenthood. Incorporating that parenthood is not feasible for everyone is fundamental to explaining the facts. We claim that the U-shaped relationship between childlessness and the education of the mother is driven by the coexistence of involuntary and voluntary causes of childlessness. Other social sciences have already discussed the definition of involuntary and voluntary childlessness (see Morgan (1991) or Toulemon (1996)). A woman will be involuntarily childless if she cannot procreate because of biological constraints leading to sterility or subfecundity; these constraints can either be innate, or acquired. We will call the first case "natural sterility" and the second

[1]Each of these groups represents a different target for the marketing literature.

[2]We divide the population into 12 categories of education as shown in Appendix A.1. Since we use U.S. data taken from the Integrated Public Use Microdata Series (IPUMS), singles are represented by the category "never-married" and married by the "ever-married, spouse present". For comments on cohabitation, see Appendix A.2.

"social sterility".[3] The definition of voluntary childlessness is more problematic; a restrictive position defines as voluntarily childless, women who have never wanted to become mothers, while a broader way to define it includes those who have never tried to become mothers. In this paper we take the broad definition.

In the model, women remain involuntarily childless when they are either naturally sterile, or, in the case of social sterility, they do not have the minimum amount of commodities needed to be able to procreate. The existence of involuntary childlessness among disadvantaged groups in the United States is described in detail by McFalls (1979).[4] He argues that lower-income groups are more exposed to causes of subfecundity than the rest of the population. Subfecundity factors that might affect the poor in developed countries are venereal diseases, malnutrition, psychopathological problems (drug abuse, stress, psychoses) and some environmental factors (pollution). The poor also have less access to quality medical services so that they are more subject to medical mistakes in abortion and cannot afford to buy fertilization services. Consequently, poor individuals are more affected by subfecundity factors because they do not have access to the same technologies. Educated women who are not naturally sterile remain childless for voluntary reasons.[5] In line with the literature, we assume that bearing and rearing a child takes time and this opportunity cost is higher for more educated women.

With this framework, we explain the U-shaped relationship between childlessness and the level of education. The relationship between these two variables is closer to a J-shaped for married women because marriage works as a life-time instrument against extreme poverty. Data from the National Survey of Family Growth for the years 1973 and 1976, which asked women questions about procreation, corroborate that, unlikely to voluntary childlessness, involuntary childlessness decreases with respect to education. Among childless women, the proportion of voluntary childlessness increases with education from 10% for women with no schooling to 70% for those with a Masters or Ph.D degree. The proportion of involuntary childlessness decreases from 70% for the lowest education group to 20% for the highest one. The proportion of women that we could not classify lies between 20 and 30 percent for all education categories. Details are given in Appendix A.3.

The model economy of this paper is composed of men and women who play a two-stage game. During the first stage, each individual is randomly matched with a partner of the opposite

[3] "Acquired sterility" means the failure to conceive after bearing a first child.

[4] The negative relationship between involuntary childlessness and income had already been documented in Wolowyna (1977), for the 1971 Canadian Census data. Romaniuk (1980) provides a good discussion of the existence of high levels of involuntary childlessness in very poor societies.

[5] With close arguments and from a cross-country analysis, Poston and Trent (1982) show the existence of a U-shaped relationship between the development level of a country and the childlessness rate.

sex and decides whether to marry or not.[6] In the second stage, couples and singles make decisions about consumption and, if they can, fertility. A single man cannot have children and consumes his life-cycle income. A single woman can either remain childless, by choice or by destiny, or become a mother and face a trade-off between consumption and fertility. When both the man and the woman decide to get married, they enter into a negotiation process in which they determine (*i*) if they have children or not, (*ii*) how many and (*iii*) how much each spouse will consume. Following a large literature initiated in the 1990s, we assume a collective cooperative negotiation process. This process is a special case of the general framework proposed by Chiappori (1988) and his co-authors (see Chiappori and Donni (2009) for a literature review) to model households' behaviors. As shown by Chiappori, this framework has considerable empirical support. In line with Iyigun and Walsh (2007), children are considered as a public good for the couple and there is no gender differences in preferences. As in Echevarria and Merlo (1999), we also assume that the time cost of rearing children is in part supported by men.

Becoming parents also entails a fixed time cost. This allows us to explain why single women choose to be mothers less often than married women, and why, when they become mothers, their fertility is almost as high as that of married mothers. Indeed, as single women cannot rely on the time input of a partner, they are less prone to pay the fixed cost of becoming a mother. However, when they do choose to pay it, they reduce the average time cost per child by having more children.

Men will want to marry because it gives them the opportunity to have children and eventually to increase their consumption. As a counterpart, they will have to give part of their time to childrearing. The advantage of marriage for women is that men alleviate the time cost of raising children and might also increase their consumption. Marriage also generates economies of scale since spouses share the expenses of household public goods. The hump shaped relationship between marriage and education is related to the high childlessness rates for extreme education levels; mainly because marrying a woman who cannot or do not want to have children is less attractive for men.

To go beyond qualitative claims, we used the U.S. Census data for the year 1990 to identify the deep parameters of the model and analyze its relevance. The model is able to reproduce the three stylized facts enumerated at the beginning. It is well known that average fertility decreases with the education of the mother, due to the higher opportunity cost of rearing children for more educated women. However, we show that Malthusian mechanisms, where

[6] To simplify, we assume the match is done randomly and that there is no second round. Results with a positive degree of assortative matching are given in Appendix C.5. We do not consider either couples of the same sex or adoption, again for simplification purposes.

education positively affects fertility, can appear for some part of the population. As this positive effect touches few individuals in the U.S., we do not observe it on an aggregate level. Concerning the U-shaped relationship between childlessness and education, we estimate that 5.0% of American women are socially sterile and 6.6% are voluntarily childless.

We also use the model to understand the changes that occurred over the period 1960-90 in marriage and fertility patterns. The model predicts that 50% of the change in marriage rates and 30% of the decrease in childlessness rates are explained by the rise in education levels. For the period 1990-2010, during which marriage rates stalled, rises in education levels explain 40% of the drop, the rest being explained by mechanisms absent from our model, such as the rise in divorce.

According to the notion of "capabilities" (Sen (1993)), fighting the causes of social sterility would allow the set of capabilities of the poor to increase. We show that reducing wage inequalities and promoting gender parity on the labor market are powerful tools to limit the proportion of involuntary childlessness generated by poverty. A drop of 5% in the Gini coefficient allows to reduce the percentage of socially sterile women by 20%.

To the best of our knowledge, this paper constitutes the first study of the determinants of marriage and childlessness in a unified framework. Gobbi (2011) models the choice to remain childless conditional on being married. She studies the determinants and the evolution of voluntary childlessness during the twentieth century. Aaronson et al. (2011) focus on a quantity-quality approach and look at how the Rosenwald Rural Schools Initiative in the early twentieth century affected fertility along both the extensive and intensive margins (childlessness versus the number of children if parent). They show that the expansion of schooling opportunities decreased the price for child quality which decreased the proportion of women with the highest fertility rates as well as childlessness rates, leading to more families of smaller size. For younger cohorts, the increase in education lead to an increase in the opportunity cost of time raising children increasing childlessness and reducing fertility. We differ from these papers by looking at the role played by involuntary childlessness and marriage opportunities.

A growing literature is concerned both with family composition and fertility choices. However, it does not allow the three facts to all be explained together. Greenwood et al. (2003) and Regalia et al. (2011) analyze both marriage and fertility decisions in a dynamic programming framework where individuals can divorce. Instead of increasing the complexity of their set-up further to allow for different motives for childlessness, we develop a model abstracting from divorce and concentrating on the mechanisms behind fertility decisions.[7] Consequently,

[7]The models of Greenwood et al. (2003) and Regalia et al. (2011) allow for voluntary childlessness, but do

our work complements preceding studies, while still replicating marriage rates in the United States for women having completed their fertility life-cycle in 1990. Our way of modeling is also different from theirs, due to our choice not to include divorce: we have a cooperative decision process inside the household while Greenwood et al. (2003) use a Nash bargaining framework and Regalia et al. (2011) use a unitary decision model where the woman chooses the number of children that the couple has.

The rest of the paper is organized as follows. Section 2 describes our stylized facts in details. The theoretical model is described in Section 3 while Section 4 displays the identification strategy for the parameters of the model and provides simulation results. Section 5 runs counter-factual experiments to understand the changes that occurred between 1960 and 1990. Conclusions are presented in Section 6.

2 Three Facts from 1990 US Census

We use the 5% sample of the U.S. Census 1990 and restrict our attention to ever-married and never-married women having completed their life-cycle fertility.[8] We look at women aged between 45 and 70 years old.[9]

Table 1 summarizes the three stylized facts we focus on in this paper. For each education category, we report the completed fertility of mothers and childlessness rate. Standard errors are given in Appendix A.5.

Fact 1: *Single women are much more likely to be childless; however, when they chose to become mothers, their fertility is lower by no more than one child compared to married mothers for all categories of education.*

The motherhood rate equals one minus the childlessness rate. As displayed in Table 1, there is a large differential in motherhood rates between single and married women. Among singles, the highest motherhood rate equals 47.6% for women with grade 11 while for married the

not look at involuntary childlessness. However Regalia et al. (2011) study how changes in relative earnings affected the increase in the proportion of single mothers.

[8]The 1990 census is the last one for the U.S. to report completed fertility. We drop from our sample women who are separated, divorced, widowed and married when their spouse is absent. The downwards relationship between fertility of mothers and education and the U-shaped relationship between childlessness and education hold for these women as well. These categories accounts for 30.5% of women. We exclude them from the sample because we do not know since when they are no longer with their partner.

[9]The oldest and the youngest women of the sample have decided to marry and to become mothers in somewhat different social and economic conditions. As shown in Appendix A.4, the facts presented in this section also hold for each five-year cohort.

Education Category	Childlessness Rates Married	Childlessness Rates Single	Completed Fertility of Mothers Married	Completed Fertility of Mothers Single	Marriage Rates Women	Marriage Rates Men
1	0.088	0.755	4.880	3.897	0.699	0.705
2	0.075	0.590	4.791	3.810	0.836	0.851
3	0.078	0.631	3.916	3.480	0.909	0.895
4	0.072	0.560	3.647	3.419	0.933	0.910
5	0.066	0.588	3.519	3.324	0.945	0.914
6	0.059	0.524	3.485	3.430	0.948	0.920
7	0.076	0.781	3.079	2.549	0.948	0.922
8	0.083	0.839	2.961	2.125	0.942	0.931
9	0.077	0.825	2.976	2.257	0.945	0.932
10	0.101	0.934	2.788	1.944	0.916	0.92
11	0.137	0.959	2.606	1.911	0.840	0.917
12	0.191	0.957	2.408	1.743	0.755	0.912
all	0.081	0.787	3.160	2.935	0.930	0.930

Table 1: Facts from U.S. Census 1990

lowest rate of motherhood is 80.9% for PhDs. On average, 78.7% of single women remain childless while 91.9% of married women become mothers.

Table 1 also shows that once single women decide to have children, they have almost the same fertility as married mothers. Around grade 11, there is almost no difference between the fertility of single and married mothers. The largest fertility differentials are observed for extreme levels of education but remain below one child. We can finally observe that, irrespective of marital status, fertility is negatively related to education. This negative relation has already been stressed in many papers without conditioning on both marital status and motherhood (see Becker (1991) pages 150-151, de la Croix and Doepke (2003) and Jones and Tertilt (2008)).

Fact 2: *Childlessness exhibits a U-shaped relationship with education for both single and married women.*

The relationship between childlessness and education is not monotonic unlike the relationship between fertility and education. We explain this U-shaped relationship by the existence of both involuntary and voluntary factors leading a woman to remain childless. For married women, the U-shaped relationship looks more J-shaped because marriage is used as a way of insuring against social sterility for women with the lowest levels of education. The increasing side of the U-shaped relationship is easy to understand: highly educated women are more likely to be childless because their opportunity cost of raising children is high.

Fact 3: *There is a hump-shaped relationship between marriage rates and education levels.*

Marriage rates are very high for intermediate levels of education: from Grade 5 to Bachelor degree, the marriage rate for women is above 90%. These rates are however much lower for extreme levels of education. Less than 70% of women with no education and around 75% of women with a PhD are married.

In Appendix A.6 we show Facts 1 and 2 for Whites, Blacks, Natives, Asians and Hispanics separately. Our facts hold for each race with one exception: white married mothers have between 1.5 and 2 times more children than singles. More details are given in Appendix A.6.

3 Theory

3.1 The Model

We consider an economy populated by heterogeneous adults, each being characterized by a triplet: sex $i = \{m, f\}$, wage w, and non-labor income a. Marriage is a two stage game: during the first stage, agents are matched randomly with an agent of the opposite sex. They decide to marry or to remain single. A match will end up in a marriage only if the two agents choose to marry. During the second stage of the game, agents decide how much to consume and how many children to have, if any. The utility of an individual of sex i is

$$u\left(c^i, n\right) = \ln\left(c^i\right) + \ln\left(n + \nu\right), \tag{1}$$

where c^i is the individual's consumption and n the number of children that he or she has, and $\nu > 0$ is a preference parameter. We chose to assume homogeneity in preferences, both across and within genders. Above the obvious analytical simplification, this is a way to measure by how much economic incentives account for our three stylized facts. Heterogenous fertility, childlessness and marriage rates among education groups are generated only from the structure of the marriage market and the heterogeneity with respect to labor and non-labor incomes. Adding heterogenous preferences for children would be a way to fill the remaining gap between the theoretical model and the stylized facts.

Each individual has a time endowment of 1 to be shared between working and child rearing. We assume that single women can have children whereas single men cannot. Having children entails time costs. First, there is a fixed cost, $\eta \in (0, 1)$ to becoming a parent. This is justified by the fact that the first child costs more in terms of time than the following children.[10] In addition to the fixed cost, there is a variable cost: each child needs $\phi \in]0,1[$ units of time to be raised. If single, the mother has to bear the full time-cost alone. When married, the husband bears a part $(1-\alpha) < 1/2$ of the childrearing time.

For simplicity, we will abstract from natural sterility until Section 3.5. But it is essential to model social sterility here, as it affects individual choices. We assume that in order to be able to give birth, a woman has to consume at least c_{m}:

$$c^f < c_{\mathrm{m}} \Rightarrow n = 0 \tag{2}$$

This assumption has been amply justified in the introduction by the fact that lower-income groups are more exposed to causes of subfecundity than the rest of the population. Notice that, unlike a good cost of children, c_{m} does not depend on the number of children that the mother will bear. A proportional good cost does not imply social sterility, as the mother could choose a low enough number of children compatible with her budget constraint.

Each adult draws a non-labor income $a^i > 0$ from a distribution $\mathcal{F}^i\left(\bar{m}_a^i, \sigma_a^i\right)$, independent of his or her education. Non-labor income corresponds to domestic production and other income uncorrelated with education. The total non-labor income for a couple equals $a^f + a^m$. On the labor market, the wages of men and women are respectively denoted w^f and w^m.

Each household has to pay a good cost, μ, which is a public good within the household. This type of cost is commonly assumed in the literature and gives some incentive to form couples (see Greenwood et al. (2012)).

Given these assumptions, the budget constraints are as follows. For single men, consumption c^m equals income minus the household good cost μ:

$$c^m = w^m + a^m - \mu$$

Single women can have children, their budget constraint is

$$c^f + \phi\left(1 + \eta(n)\right) w^f n = w^f + a^f - \mu \tag{3}$$

[10] Turchi (1975) estimates that the mean number of hours spent per day on childrearing for a one-child family is 1.4, for the first 18 years. For a two-child family it is 0.99 per child. And for a three-child family, it is 0.93.

where

$$\eta(n) = \begin{cases} \dfrac{\eta}{n} & \text{if } n > 0 \\ 0 & \text{if } n = 0. \end{cases} \tag{4}$$

Given the time constraint $\phi(1+\eta/n)n \leq 1$ the maximum number of children a single woman can have is $\bar{n}_{\mathrm{M}} = \frac{1-\phi\eta}{\phi}$. For couples, total non-labor income of the household net of cost is $a = a^m + a^f - \mu$. Their budget constraint is

$$c^f + c^m + \phi\,(1+\eta(n))\left(\alpha w^f + (1-\alpha)w^m\right)n = w^m + w^f + a \tag{5}$$

The maximum fertility rate of a married woman equals $n_{\mathrm{M}} = \frac{1-\alpha\phi\eta}{\alpha\phi}$. This is greater than the maximum fertility of a single woman ($n_{\mathrm{M}} > \bar{n}_{\mathrm{M}}$) because husbands help in the raising of children. If spouses share the childrearing cost equally, the maximum number of children a woman can have equals $(2-\phi\eta)/\phi$.

To model couples' decision making, we assume a cooperative collective decision model following Chiappori (1988).[11] Spouses negotiate on c^m, c^f and n. Their objective function is

$$U(c^f, c^m, n) = \theta(w^f, w^m)\; u(c^f, n) + (1 - \theta(w^f, w^m))\; u(c^m, n)$$

where $\theta(w^f, w^m)$ is the wife's bargaining power that depends on education and is given by

$$\theta(w^f, w^m) \equiv \frac{1}{2}\,\underline{\theta} + (1-\underline{\theta})\frac{w^f}{w^f + w^m} \tag{6}$$

We specifically assume that the negotiation power of spouses is bounded, with a lower bound equal to $\underline{\theta}/2$, and positively related to their relative wage. We use education as a proxy for potential wages in our model so that negotiation power is in fact positively related to the relative education of the spouse.[12] The boundedness of the bargaining power function comes from the legal aspect of marriage: spouses have to respect a minimal level of solidarity inside marriage.

The different assumptions we have introduced imply some advantages to being married. It

[11] Due to the presence of the fixed cost μ, there is always a surplus coming from marriage. By adopting a collective cooperative decision model, rather than a Nash bargaining process where potential spouses share the marriage surplus, we avoid marriage rates being equal to one. With a Nash bargaining process, and no frictions in the marriage market, everybody would get married in order to share the surplus. In this type of framework, the only way to allow for some proportion of singles would be to assume some negative shocks on the quality of the matching.

[12] See de la Croix and Vander Donckt (2010) for a discussion. An alternative consists in assuming a negotiation power that depends on the spouses' relative labor income rather than their relative education as in Iyigun and Walsh (2007). We could also have included non labor income in the bargaining power, as advised by Pollak (2005).

allows the cost of the household μ to be shared. Being married is the only way for men to have children. It allows women to reduce the opportunity cost of children (as long as $\alpha < 1$). Marriage gives at least one of the partners the opportunity to increase his/her consumption compared to the situation where he or she remains single.

We impose the following assumptions on the parameters.

Assumption 1 *The fixed cost η is not too large:*

$$\nu(1 - c_m) > \eta \quad , \quad \frac{-\sqrt{\eta} + \sqrt{\nu}}{\phi\sqrt{\eta}(\nu - \eta)} > 1.$$

3.2 Possible "Regimes"

We solve the problem backwards, first considering the consumption and fertility choice conditional on being married or not. Before studying (in Subsection 3.3) the precise conditions on wages and non-labor income under which the various constraints bind, we list the different possible "regimes", each describing a living arrangement.

3.2.1 Single Women

A single woman maximizes (1) subject to (2), (3), (4) and $0 \leq n \leq \bar{n}_{\mathrm{M}}$. These women can be in different regimes, depending on which constraint is binding.

Regime I. (Interior solution) If no other constraint than the budget constraint is binding and the corner solution with no children does not dominate, the interior regime prevails with

$$\begin{aligned} c_{\mathrm{I}}^f &= \frac{w^f(1 + \phi(\nu - \eta)) + a^f - \mu}{2} \\ n_{\mathrm{I}} &= \frac{1}{2}\left[\frac{w^f(1 + \phi(\nu - \eta)) + a^f - \mu}{\phi w^f}\right] - \nu. \end{aligned} \tag{7}$$

Equation (7) shows that the effect of woman's education on fertility is ambiguous. A higher w^f implies the usual income effect which raises fertility. But the increase in w^f also raises the time cost to rear children, which reduces fertility through a substitution effect. If $a^f > \mu$, the substitution effect dominates and n_{I} decreases with w^f, while the reverse is true if $a^f < \mu$.

Regime II. (Involuntary childlessness) If her income does not allow the consumption level required to have children to be reached, the single woman lives in poverty and is prevented from freely choosing the number of children that she has: $c^f_{\text{II}} = w^f + a^f - \mu < c_{\text{m}}$ and $n_{\text{II}} = 0$.

Regime III. (Get fit to procreate) If her income is sufficiently high to escape Regime II but not high enough to have as many children as in Regime I, it may be optimal for the single woman to work and consume more in order to be able to procreate: $c^f_{\text{III}} = c_{\text{m}}$ and

$$n_{\text{III}} = \frac{w^f(1-\phi\eta) + a^f - c_{\text{m}} - \mu}{\phi w^f}.$$

A necessary condition for this regime to prevail is $a^f < c_{\text{m}} + \mu$. Otherwise the woman would be able to consume more than c_{m} without working. $a^f < c_{\text{m}} + \mu$ implies that n_{III} increases with the wage w^f. Regime III echoes Malthus's fertility theory (see e.g. Galor (2005)).

Regime IV. (Voluntary childlessness) When being childless yields the highest utility, the voluntary childlessness regime prevails: $c^f_{\text{IV}} = w^f + a^f - \mu$ and $n_{\text{IV}} = 0$. This regime will prevail when the opportunity cost of having children is high and the single woman fully specializes in labor market activities.

Regime V. (Maximum fertility) When the opportunity cost of having children is low and the non-labor income is high, it may be optimal for the woman to fully specialize in the production of children: $c^f_{\text{V}} = a^f - \mu$ and $n_{\text{V}} = \bar{n}_{\text{M}}$.

3.2.2 Couples

The problem for the couple is

$$\max_{c^f, c^m, n} \theta(w^f, w^m) \ln c^f + (1 - \theta(w^f, w^m)) \ln c^m + \ln(\nu + n)$$

subject to (2), (4), (5) and $0 \leq n \leq n_{\text{M}}$.[13] Six living arrangements (regimes) are possible.

[13] An alternative would be to add two participation constraints: $u^f(\textit{married}) \geq u^f(\textit{single})$ and $u^m(\textit{married}) \geq u^m(\textit{single})$. This implies that the partner with the highest negotiation power can have an interest in reducing his/her welfare in order to incite his/her match to accept marriage. However, because rationality is common knowledge, such a marriage contract would not be credible: as couples cannot divorce, the partner with the highest negotiation power has an incentive to deviate from the agreement.

Regime VI. (Interior solution) If no constraint is binding and the corner solution with no children does not dominate we have:

$$\begin{aligned}
c^m_{\text{VI}} &= \frac{\left(1-\theta(w^f,w^m)\right)\left[w^f\left(1+\phi\alpha(\nu-\eta)\right)+w^m\left(1+\phi(1-\alpha)(\nu-\eta)\right)+a\right]}{2} \\
c^f_{\text{VI}} &= \frac{\theta(w^f,w^m)\left[w^f\left(1+\phi\alpha(\nu-\eta)\right)+w^m\left(1+\phi(1-\alpha)(\nu-\eta)\right)+a\right]}{2} > c_{\text{m}} \\
n_{\text{VI}} &= \frac{w^f\left(1+\phi\alpha(\nu-\eta)\right)+w^m\left(1+\phi(1-\alpha)(\nu-\eta)\right)+a}{2\phi\left(\alpha w^f+(1-\alpha)w^m\right)} - \nu > 0.
\end{aligned}$$

An increase in the wage of the husband has an ambiguous effect on fertility:

$$\frac{\partial n_{\text{VI}}}{\partial w^m} > 0 \Leftrightarrow w^f > \frac{1-\alpha}{2\alpha-1}a.$$

In most of the literature (Galor and Weil (1996), etc.) $\alpha = 1$, then an increase in the wage of the husband has a pure income effect and $\partial n_{\text{VI}}/\partial w^m > 0$. When $\alpha < 1$, the income effect always dominates the substitution effect in families with very low non-labor incomes ($a < 0$). In families with higher non-labor incomes ($a > 0$), the substitution effect dominates only when the wage of the woman is low relative to the non-labor income of the couple.

In the case of an increase in the wife's wage, we find:

$$\frac{\partial n_{\text{VI}}}{\partial w^f} < 0 \Leftrightarrow w^m > \frac{\alpha}{1-2\alpha}a.$$

As long as $\alpha > 1/2$, an increase in the woman's wage always reduces fertility when $a > 0$. In families with very low non-labor income ($a < 0$), the income effect dominates if the wage of the husband is also very low.[14]

Finally, n_{VI} does not depend on the negotiation power $\theta(w^f, w^m)$ as there are no gender differences in preferences for children.

Regime VII. (Involuntary childlessness) When total net income is too low to guarantee a sufficient consumption level to procreate: $n_{\text{VII}} = 0$,

$$c^m_{\text{VII}} = \left(1-\theta(w^f,w^m)\right)\left(w^m+w^f+a\right) \quad \text{and} \quad c^f_{\text{VII}} = \theta(w^f,w^m)\left(w^m+w^f+a\right) < c_{\text{m}}.$$

Regime VIII. (Eat and procreate) When the wife's bargaining power is too low to guarantee a sufficient consumption level allowing her to procreate in the interior regime, it

[14] Jones and Schoonbroodt (2010) provide a discussion on the possible impacts of women's wages on fertility.

can be optimal for the husband to give up some consumption in order to have children. Then $c^f_{\text{VII}} = c_{\text{m}}$,

$$\begin{aligned}
c^m_{\text{VII}} &= \frac{\left(1-\theta(w^f,w^m)\right)\left[w^f\left(1+\phi\alpha(\nu-\eta)\right)+w^m\left(1+\phi(1-\alpha)(\nu-\eta)\right)+a-c_{\text{m}}\right]}{2-\theta(w^f,w^m)} \\
n_{\text{VII}} &= \frac{1}{2-\theta(w^f,w^m)}\left[\frac{w^f\left(1+\phi\alpha(\nu-\eta)\right)+w^m\left(1+\phi(1-\alpha)(\nu-\eta)\right)+a-c_{\text{m}}}{\phi\left(\alpha w^f+(1-\alpha)w^m\right)}\right]-\nu.
\end{aligned}$$

This is the only regime in which fertility depends on the wife's bargaining power. This makes fertility subject to three effects: the usual income and substitution effects and a negotiation power effect. The latter arises because an increase in the wife's education gives her more bargaining power. As, in this regime, her consumption is fixed, the only way to obtain more utility is through increased fertility. The net effect on fertility is the result of these three forces.

Regime IX. (Voluntary childlessness) When choosing to be childless yields the highest utility, we have: $n_{\text{IX}} = 0$,

$$c^m_{\text{IX}} = \left(1-\theta(w^f,w^m)\right)\left(w^m+w^f+a\right) \quad \text{and} \quad c^f_{\text{IX}} = \theta(w^f,w^m)\left(w^m+w^f+a\right).$$

In this case, the wage of the wife is so high that rearing children is too expensive. Both spouses fully specialize in labor market activities.

Regime X. (Eat and procreate to the maximum) When it is optimal for the husband to give up some consumption for his wife to specialize entirely in procreation: $c^f_{\text{X}} = c_{\text{m}}$, $n_{\text{X}} = n_{\text{M}}$ and

$$c^m_{\text{X}} = \left(\frac{2\alpha-1}{\alpha}\right)w^m + a - c_{\text{m}}.$$

Compared to Regime VIII, the optimal fertility rate of the couple does not depend on the negotiation power of the wife. Indeed, as she has already reached her maximal fertility rate, she is not able to give birth to an additional child to increase her utility.

This regime does not exist for single woman as it relies on the additional income provided by the husband which allows uneducated mothers to specialize in the production of children and still consume c_{m}.

Regime XI. (Maximum fertility) When the couple is sufficiently rich for the wife to specialize in childrearing without requiring any sacrifice from the husband, $n_{\mathrm{XI}} = n_{\mathrm{M}}$,

$$c^m_{\mathrm{XI}} = \left(1-\theta(w^f, w^m)\right)\left(\left(\frac{2\alpha-1}{\alpha}\right)w^m + a\right) \quad \text{and} \quad c^f_{\mathrm{XI}} = \theta(w^f, w^m)\left(\left(\frac{2\alpha-1}{\alpha}\right)w^m + a\right).$$

3.3 Fertility and Consumption Choices

Conditional on a woman being single, we can determine optimal fertility and consumption as a function of her wage and non-labor income.

In Appendix B.1, we define non-labor income thresholds, $\underline{a}$ and $\overline{a}$, and wage thresholds, $\mathcal{W}^f_0(a^f), \mathcal{W}^f_1(a^f)$, $\mathcal{W}^f_2(a^f)$,$\mathcal{W}^f_3(a^f)$,$\mathcal{W}^f_4(a^f)$ and $\mathcal{W}^f_5(a^f)$. We also provide and prove a proposition which allows us to describe the optimal behavior of a single woman endowed with (w^f, a^f). In Figure 1 and the following paragraphs we describe the main results of this proposition.

When $a < \underline{a}$, a single woman is childless no matter her wage. This is the case when her non-labor income is too small. In this case, the wage allowing her to procreate is higher than the wage for which she would choose to be voluntarily childless. This implies that once she can afford a child, the time spent with him/her becomes too expensive.

In Figure 1, we represent the relationship between the fertility of a single woman and her wage (education), when $a > \underline{a}$. The left panel shows a non-monotonic relationship between wage and fertility. The net non-labor income $(a^f - \mu)$ is still not high enough to cover the minimal amount of consumption needed to procreate. For a low w^f, Regime III (get fit to procreate) prevails and an increase in the wage of the woman increases her fertility. As her consumption is fixed at level c_{m} in this regime, fertility is positively related to her wage until it is high enough to reach the interior regime (I). In the interior regime, n_{I} decreases with w^f because the substitution effect dominates $(a^f \geq \underline{a} > \mu)$.

In the right panel, non-labor income is high enough to ensure a consumption c_{m} for any fertility level, even in the absence of labor income. As a consequence, Regime III disappears, and Regime V becomes possible. Regime V is a corner solution where the wage of the woman is so small that it is optimal for her to specialize in childrearing and consume her net non-labor income $a^f - \mu$.

On the whole, Figure 1 shows that uneducated single women will either be involuntarily childless or in the maximum fertility case, this will depend on their non-labor income. Conversely, highly educated women will be voluntarily childless (Regime IV) because the opportunity

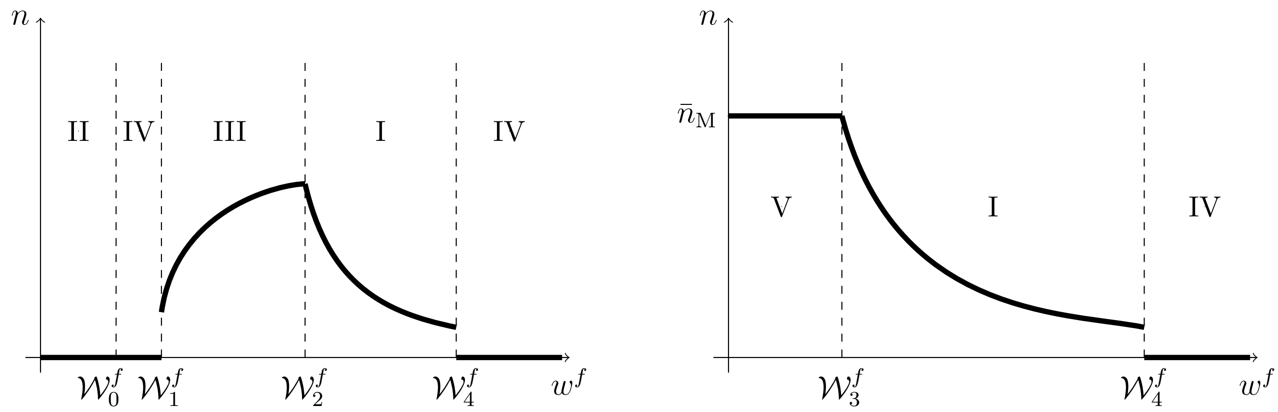

Figure 1: Fertility Conditional on Being Single when $a^f \in [\underline{a}, \overline{a}[$ and when $a^f \geq \overline{a}$

cost of having children (non-participation to the workforce) is high. This figure also shows another important feature: the existence of a fixed cost of becoming a parent implies that the passage from being childless to being a parent cannot be represented by a continuous function. The fixed cost makes it never optimal to have a very low number of children.

We now turn our attention to couples. To simplify the computations, we expose the limit case $w^m = 0$, but all the results remain valid by continuity for small w^m. In Appendix B.2, we define non labor income thresholds A_0, A_1, A_2, A_3, A_4 and A_5 as well as wage thresholds $\mathcal{W}^f_A(a)$, $\mathcal{W}^f_A(a)$, $\mathcal{W}^f_A(a)$, $\mathcal{W}^f_A(a)$, $\mathcal{W}^f_A(a)$, $\mathcal{W}^f_B(a)$, $\mathcal{W}^f_{\underline{C}}(a)$, $\mathcal{W}^f_{\overline{C}}(a)$, $\mathcal{W}^f_D(a)$, $\mathcal{W}^f_E(a)$, $\mathcal{W}^f_F(a)$, $\mathcal{W}^f_G(a)$ and $\mathcal{W}^f_H(a)$. In Appendix B.2, we provide and prove a proposition that describes the optimal behavior of a couple endowed with (w^f, a). In Figures 2, 3 and 4 and the following paragraphs, we describe the main results of this proposition.

As for a single woman, for very low non-labor income ($a < A_0$) and, by extension, low w^m, a couple will be childless whatever the wife's wage. The wage allowing the couple to have children is higher than the wage for which they would choose to be voluntarily childless (Regime IX).

For higher levels of non-labor income ($a > A_0$), having children becomes feasible and optimal for some wages. Figure 2 (left) shows that, when $a \in]A_0, A_1]$, a couple can have children in the eat and procreate regime (Regime VIII) but the interior regime (Regime VI) is never optimal. In Regime VIII, the husband voluntarily reduces his consumption in order to enable his wife to have children. As shown in Figure 2 (right), Regime VI becomes optimal for some wages and $a > A_1$. In Figure 2 , the negotiation power effect dominates the income and substitution effects in the eat and procreate regime. Then, an increase in w^f increases fertility in Regime VIII.

$\mathcal{W}^f_A$ corresponds to the wage allowing couples to procreate. For a wage greater than but close

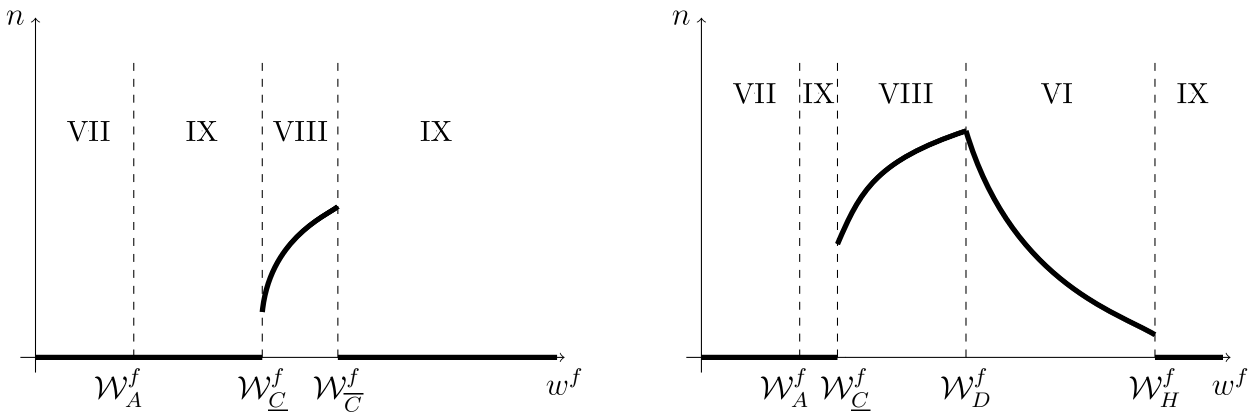

Figure 2: Fertility Conditional on Being Married when $a \in]A_0, A_1]$ and when $a \in]A_1, A_2]$

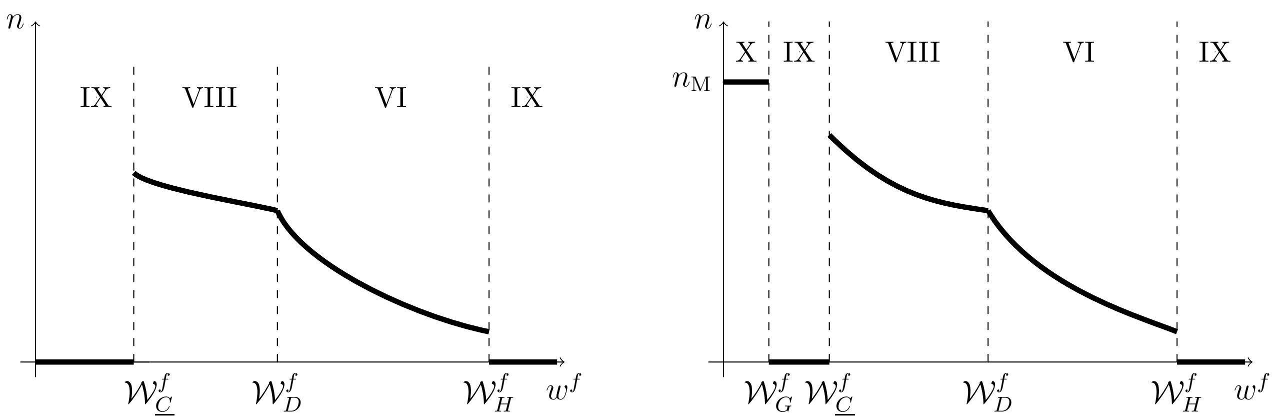

Figure 3: Fertility Conditional on Being Married when $a \in]A_2, A_3]$ and when $a \in]A_3, A_4]$

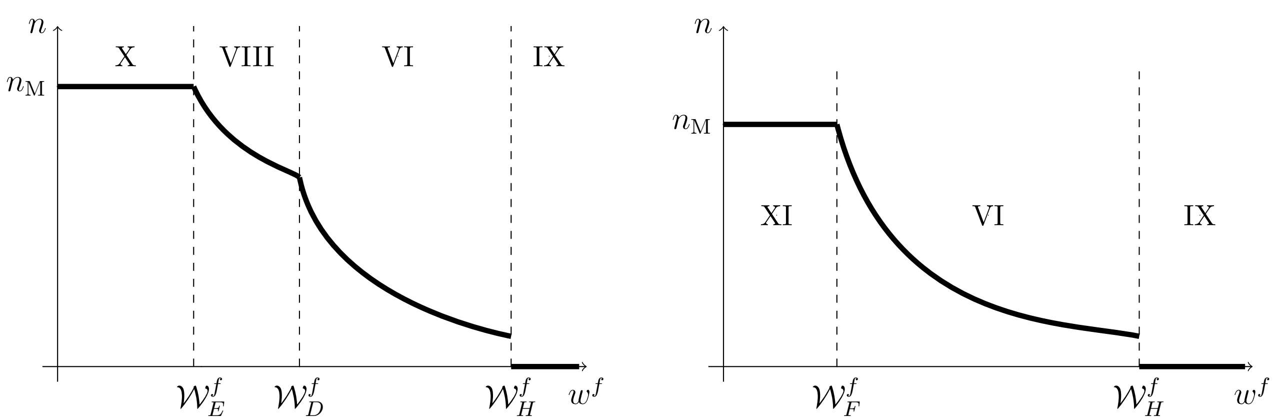

Figure 4: Fertility Conditional on Being Married when $a \in]A_4, A_5]$ and when $a > A_5$

to $\mathcal{W}_A^f$, the consumption of the husband will be close to zero, implying that parenthood is not optimal. This explains why Regime IX always precedes Regime VIII in Figure 2 and Figure 3 (left).

In Figures 3 and 4, $a > c_{\mathrm{m}}$. Involuntary childlessness disappears as a couple can always ensure that the wife can consume c_{m}. It also implies that fertility decreases with women wages in Regime VIII because the substitution effect dominates.

Regimes X and XI where the wife has the highest fertility appear in Figures 3 (right) and 4 for low wages. In Regime X, the wife consumes c_{m} and entirely specializes in childrearing. This regime becomes feasible once $a > A_2$, but is optimal for some wages only when $a \geq A_3$ (when $a < A_3$ the consumption of the husband is too low). Once w^f becomes sufficiently high, the wife still consumes c_{m} but spends a part of her time on labor market activities. The additional income earned by the couple is allocated to the consumption of the husband. Finally, Figure 4 (right) exhibits the same fertility pattern as Figure 1 for a single woman and the same intuitions apply.

Poor couples will then be either childless (involuntarily or voluntarily) or in a regime where the wife has the highest fertility. In all cases, couples in which the wife is highly educated remain voluntarily childless.

3.4 Marriage Decisions

We now consider the decision to accept a marriage offer from a randomly drawn person of the opposite sex. Once this potential partner has been drawn, we know the vector (w^f, a^f, w^m, a^m). This allows each individual to compare his/her utility as a single and as a married person and decide whether to accept marriage or not. In this section, we provide some elements to highlight this choice.

Let us first consider the potential bride. From Propositions in Appendices B.1 and B.2 we know the two relevant regimes she has to compare. When the potential bride has both a low wage and a low non-labor income, she would live either in Regime II or Regime III if she remains single. If the relevant regime for the woman when single is Regime II, then she always prefers to be married with a husband allowing her to consume at least $c^{\min}$, as

$$u(c^f_{\mathrm{II}}, 0) < \min\{u(c_{\mathrm{m}}, n_{\mathrm{VIII}}), u(c^f_{\mathrm{VII}}, n_{\mathrm{VII}}), u(c^f_{\mathrm{IX}}, 0), u(c_{\mathrm{m}}, n_{\mathrm{M}}), u(c^f_{\mathrm{XI}}, n_{\mathrm{M}})\}.$$

She would reject a marriage offer only in the case where the couple would be in Regime VII and if $u(c^f_{\mathrm{I}}, 0) > u(c^f_{\mathrm{VII}}, 0)$, that is,

$$w^f + a^f - \mu > \theta(w^f, w^m)(w^m + w^f + a).$$

The only motive for a very poor women to marry a men who does not provide enough to allow her to procreate lies in the scale economies allowed by marriage: instead of paying a cost μ to live as single people, they pay the same cost to live as a couple. Then, an uneducated women will accept to marry an uneducated men because similar education ensures that negotiation powers remain close to one half which allows the surplus from marriage to be shared equally.

If the relevant regime for a woman when single is Regime III, she always prefers to be a single mother who consumes c_{m} rather than a childless married woman who consumes less than c_{m} (i.e. $u(c_{\mathrm{m}}, n_{\mathrm{III}}) > u(c^f_{\mathrm{VII}}, 0)$). Hence, she would always reject marriage offers leading to VII. Moreover, $u(c_{\mathrm{m}}, n_{\mathrm{III}}) < \min\{u(c_{\mathrm{m}}, n_{\mathrm{M}}), u(c^f_{\mathrm{XI}}, n_{\mathrm{M}})\}$, since in Regimes X and XI the woman would consume at least c_{m} and would have the maximum number of children. So she would always accept marriage offers leading to X and XI.

For higher w^f and a^f, Regimes I, IV or V are accessible as a single person. We can give some details in two cases. First, Regime V (maximum fertility) is always preferred to Regime VII and VIII: a poor woman (low w^f) will always prefer to be single with the maximum number of children rather than being an involuntary childless married woman or a married woman who has to be fed in order to procreate. And second, Regime IV (voluntary childlessness) does not dominate in only two circumstances: (a) when the additional income coming from the man is not high enough to incite the woman to become a mother but sufficiently close to her own income for the marriage surplus to be shared (Regime IX) and (b) when the man's wage is high enough to incite her to become a mother (Regime VI or XI).

We can then conclude that, unlike rich women, poor women will accept almost any match on the marriage market. This highlights the role of marriage as an institution protecting women against poverty, involuntary childlessness and even against living in a "get fit to procreate" regime. Richer women do not need to be protected against involuntary childlessness by marriage. On the contrary, for a rich woman, being matched with a rich man is the occasion to have children rather than being a voluntarily childless single person because marriage reduces the opportunity cost of motherhood.

As being single is not always a personal choice, let us now turn to the problem of the potential husband. A single man has the following indirect utility,

$$\bar{u}^m = \ln(w^m + a^m - \mu) + \ln \nu.$$

Getting married is the only way for him to become a father but entails two costs: an opportunity cost due to the time spent with children and a potential decrease of his consumption. This implies that a man will agree to marry without having children only if it increases his own consumption, which arises when the potential bride is rich enough. Regimes VII and IX can then dominate singleness for the man.

The opportunity to become a father is not always sufficient to incite the man to marry: if having children decreases his consumption too sharply, he will remain single (singleness would dominate Regimes VI, VIII, X and XI). This could apply to a man with a low income who is matched with a woman who would be involuntarily childless as a single person: in order to have children in the "eat and procreate" regime, he would have to give up too much of his income and consume too little.

Once again, marriage is a protection against involuntary childlessness: two partners could have children by pooling their income while they would have remained childless and single otherwise. Men who live in Regime VIII agree to reduce their consumption in order to enable their wives to have children. This is optimal if c_{m} is not too large and the man not too poor. Marriage can even prevent poor women becoming poor mothers. This is the case when a rich man agrees to marry a poorly educated woman: despite her negotiation power being minimal, she will consume more than c_{m} because the consumption of rich men's wives increases linearly with w^m.

Parameter α is important for marital decisions. For high values of α, the model predicts higher marriage rates for rich men than for rich women as richer men have more reasons to marry: they can have children at a low opportunity cost. For rich women the incentive to marry is now low and marrying a poor man would only entail costs (they would remain childless whatever the marital status). Poor women, on the contrary, will marry more often as they are less often rejected by rich men who can have children without reducing their consumption too much. For low values of α, the opportunity cost of becoming a father is high and nothing ensures that rich men marry more often than rich women.

The model predicts a positive degree of assortativeness due to parameter $\underline{\theta}$. A higher $\underline{\theta}$ implies that spouses' incomes are more equally shared. An individual with a much larger income than his or her potential spouse will then be more likely to reject the offer as he/she would have to give up too much of their own consumption.

To conclude, the model predicts lower marriage rates for extreme social classes. Women with low levels of education are more likely to be involuntarily childless while highly educated women will probably want to remain childless. This makes women from extreme social classes less attractive to men. Marriage could however occur when wages and non-labor incomes are sufficiently close for them to share the marriage surplus.

3.5 Natural Sterility

So far we have abstracted from natural sterility. This amounts to assuming that involuntary childlessness is only due to bad lifestyle and low income. In reality, there exists a positive degree of sterility among men and women that is uncorrelated to lifestyle. We take this into account here.

We assume that natural sterility is uniformly distributed over education categories. Let $\chi \in [0,1]$ and $\zeta \in [0,1]$ respectively describe the percentage of female and male who are naturally sterile. $\mathrm{E}u(c^i, n)$ denotes the expected utility of an agent of sex i and $\mathrm{E}U(c^f, c^m, n)$ the expected utility of a couple. We have that $\mathrm{E}u(c^m, n) = \overline{u}^m$ and:

$$\begin{aligned}\mathrm{E}u(c^f, n) &= \chi \cdot u(w^f + a^f - \mu, 0) + (1-\chi) \cdot u(c^f, n) \\ \mathrm{E}U(c^f, c^m, n) &= (\chi + (1-\chi)\zeta) \cdot U(c^f, c^m, 0) + (1 - \chi - (1-\chi)\zeta) \cdot U(c^f, c^m, n).\end{aligned}$$

We explicitly assume that single women are not concerned with male sterility as they can have multiple sexual partners.[15] It implies that married women are more concerned with sterility as they can be matched with a sterile partner. The marriage game now has three stages: first, people enter the marriage market ignoring their sterility status and decide whether or not to marry the partner they have been matched; second, they discover their sterility status at no cost; and third, they decide how much to consume and, eventually, how many children to have. As other assumptions have not changed, we obtain the following proposition:

Proposition 1 *When a single woman is naturally sterile, $c^f = a^f + w^f - \mu$, and $n = 0$. When a single woman is not naturally sterile, her optimal fertility choices are described by Proposition 1. When a couple is not naturally sterile, spouses' optimal decisions are described by Proposition 2; while, when a couple is naturally sterile, spouses share the net income of the household such that $n = 0$,*

$$c^m = \left(1 - \theta(w^f, w^m)\right)\left(w^m + w^f + a\right) \quad \textit{and} \quad c^f = \theta(w^f, w^m)\left(w^m + w^f + a\right).$$

[15] A woman who has n different sexual partners in her life meets only infertile partners with a probability ζ^n. If natural sterility among males is 5% and a woman has only two different partners in her lifetime, she has a probability of 0.0025 of meeting only infertile partners. According to the National Survey of Family Growth (2002), the average number of lifetime sex partners for women who have always been single in the U.S. is 7.44.

Proof. As discovering sterility is made at no cost, choices of fecund single women and fecund couples are the same as in the benchmark model. For those who discover that they are naturally sterile, $n = 0$, implying that: (i) single women consume their net income and (ii) couples share their net income as an involuntarily childless couple (Regime VII). ∎

Compared to the benchmark model, we have added two categories of agents: sterile single women and sterile couples. The possibility of being naturally sterile or being matched with a naturally sterile partner has an impact on the incentive to marry. As couples face a higher risk of being sterile than single women, natural sterility tends to reduce the overall marriage rate compared to the benchmark model. In particular, the number of marriages between highly educated men and poorly educated women should be strongly reduced. Indeed, the only incentive for rich men to marry a poor woman lies in the possibility of becoming a father. Now, when they marry a poor woman, they have to reduce their consumption *and* to take the risk of remaining childless. Women with high education levels are less concerned by this phenomenon as they can have children on their own.

4 Identification of Parameters and Simulations

The theory we developed in the previous section is very parsimonious. We have only seven preference and technological parameters, to which we add two parameters for the distribution of non-labor income. Despite this parsimony, we will match the three facts of Section 2. However, once we compare our predictions to data for moments we do not try to replicate, our model behaves well qualitatively but not quantitatively. From this perspective, adding more features to the model, such as exogenous assortative matching and other dimensions of inequality would help, but at the cost of tractability.

4.1 Identification

We identify the parameters of the model using the Simulated Method of Moments (SMM). The nine parameters, listed in Table 2, are identified by minimizing the distance between 48 empirical and simulated moments. These moments are the completed fertility of mothers and childlessness rates among both singles and married women, for the 12 education categories listed in Table 5. The objective function to minimize is given by:

$$f(p) = [d - s(p)]\,[W]\,[d - s(p)]'$$

where p is the vector of model's parameters, d denotes the vector of empirical moments and s the vector of simulated moments, depending on the parameters. W is the optimal weighting matrix, i.e. the inverse of the variance-covariance matrix of the empirical moments (Duffie and Singleton (1993)). In our case, this simplifies into a diagonal matrix composed of the inverse of the statistical standard errors of the empirical moments. Indeed, the covariances across various education categories are zero, as these categories are independent (notice also that they contain different numbers of observations). Moreover, the covariances between moments for married people and single people are also necessarily zero. Finally, the covariance between childlessness rates and fertility is also zero as childlessness is nil for all individuals for which $n > 0$. Intuitively, the use of the optimal weighting matrix implies that moments with the lower standard errors will have a higher weight. Consequently, for these moments, a higher distance between the data and the simulated moments will be more heavily penalized in the objective function.

In order to construct the vector of simulated moments, we need some assumptions. First, we assume that, for each category of education, the non-labor income of women follows a log-normal distribution of mean κ_a and variance σ_a^2, from which we draw T observations. Writing

$$\kappa_a = \ln(\bar{m}_a \bar{w}) - \frac{\sigma_a^2}{2}$$

where $\bar{w}$ denotes women's average wage, the parameter $\bar{m}_a$ can be interpreted as the average ratio of non-labor income to labor income.

Wages are exogenous and computed as follows:

$$w_e = \gamma \exp\{\rho e\} \tag{8}$$

where e denotes the average number of years of education in each category (Table 5). We assume that the gender wage gap γ is the same across education categories[16] and equal to 0.9 (Erosa et al. (2010)) and we also assume that the Mincerian "rate of return to schooling" ρ is equal to 0.1 (Krueger and Mikael (2001)). Normalizing wages with respect to the wage of a woman who has a doctoral degree, the maximum wage among men equals 1.111 while the minimum wage among women equals 0.135. The main drawback of the Mincerian approach is to assume the same return to schooling for all schooling levels; its asset is to let income depend on two parameters only, the gender wage gap γ and the return to schooling ρ, which can be the subject of counterfactual experiments. Such counterfactual exercises are provided

[16] Assuming that γ increases with education, as some studies in the 1980s suggested, modifies the results only marginally. Lips (2003) shows that this "education penalty" almost disappeared during the 1990's.

in Subsections 5.3 and 5.4.

The two sterility parameters are fixed using information from the literature on natural sterility. First we set the percentage of naturally sterile couples, $\chi + (1 - \chi)\zeta$, at 3.7%. This number is obtained from Leridon (2008) who uses the Henry database, which consists of data for rural France from the 18th century, when fertility control was ineffective. Restricting the sample to couples where the husband and wife were still living together at age 50, 3.7 % of women who married at age 20-24 remained childless, which gives the rate of natural sterility for couples. Second, our reading of the literature on the prevalence of fertility problems is that roughly half of them with a diagnosed cause are related to the man, and half to the woman.[17] We therefore set χ equal to ζ. The two restrictions lead to $\chi = \zeta = 1.87\%$.

To compute simulated moments, we proceed as follows. For each woman in each category of education, we draw a potential husband from the empirical distribution of education levels among men.[18] For each level of men's education, non-labor income is drawn from the same distribution as women. Then, for each category of education for women, we obtain T decisions about marriage and fertility and we are able to calculate the simulated moments.

The minimization of the objective function $f(p)$ was first run using PIKAIA and the results used as initial values in UOBYQA. PIKAIA is a genetic algorithm developed by Charbonneau (2002), it allows global extrema to be found in highly non-linear optimization problems where there exist a high number of local extrema. We used PIKAIA in a first step to identify the region in the parameter space where the global maximum lies. Once this region has been identified, we used a faster algorithm called UOBYQA (Powell (2002)) designed to find a maximum of a well-behaved problem. It was developed for optimization when first derivatives of the objective function were not available and takes account of the curvature of the objective function, by forming quadratic models by interpolation. We ran these two algorithms in FORTRAN 90. In the numerical implementation, we also assume that the number of births is an integer rather than a continuous variable. This simplification does not alter the main mechanisms at play but simplifies computational exercises (and is also realistic).

[17] This estimation is quite imprecise as definitions of sterility and infertility used in the literature lack uniformity (see Gurunath et al. (2011)).

[18] We rescaled the empirical marriage rate of men in order to have an equal number of men and women.

Description	Parameter	Value	Standard error
Variance of the log-normal distribution	σ_a	0.312	0.0098
Average ratio of non-labor income to labor income	$\bar{m}_a$	0.848	0.0109
Preference parameter	ν	6.683	0.1090
Minimum consumption level to be able to procreate	c_{m}	0.311	0.0046
Good cost to be supported by a household	μ	0.328	0.0066
Bargaining parameter	$\underline{\theta}$	0.574	0.0082
Fraction of childrearing to be supported by women	α	0.597	0.0041
Time cost of having children	ϕ	0.224	0.0022
Fixed cost of children	η	0.201	0.0081

Table 2: Identified Parameters, $T = 100,000$

4.2 Results

The identified parameters are listed in Table 2. Appendix C.1 explains how the standard errors are computed.[19] Assumptions 1 and 2 hold under the values presented in Table 2.

Non-labor income amounts on average to 84.8% of labor income. This number seems quite high, unless we interpret the non-labor income as including, in addition to home production, bequests, capital income and transfers (including social security) that are not correlated with the education level of the recipient. To have an idea of the magnitude of σ_a, we computed a Gini coefficient on women's simulated life-cycle income, $w^f + a^f$, which came 0.166. The estimated σ_a is relatively on the low side, but this is not surprising as some dimensions of inequality are absent from the model, such as wage dispersion for similar education levels.

To interpret the value of c_{m} and μ, remember that wages vary on a scale from 0.135 to 1 for women. The wage of a woman having completed Grade 9 equals 0.333 so that women with a degree lower than Grade 10 are not able to pay μ with their labor income. A single woman with the lowest wage will need a non-labor income higher than 0.639 not to be involuntarily childless.

Parameter $\underline{\theta} = 0.574$ means that the minimum negotiation power of a spouse is $\underline{\theta}/2 = 0.287$. The childrearing time is shared between spouses. We estimate that men are involved for 40.3% of this time. The values for ϕ and η imply that the first child costs $\phi(1 + \eta) = 26.9\%$ of the time endowment of one parent, while the second child costs 22.4%. Following the

[19]For curiosity, we have identified the parameters under the assumption that they are race specific, and that marriage markets are segmented by race. Results are in Appendix C.2. To be complete, we provide in the same appendix the identification after having removed from the sample the "disables". A discussion of whether one should include them or not is provided in Appendix A.7.

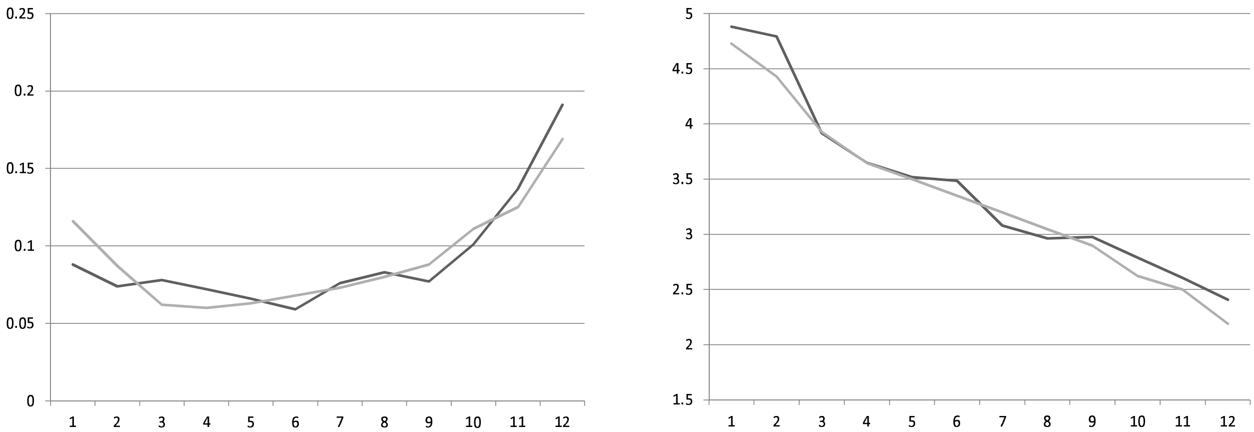

Figure 5: Childlessness Rate and Completed Fertility of Mothers, by Education Categories, Married Women. Data (black) and Simulation (grey)

values of α, η and ϕ, the maximum number of children that a married woman can raise is seven, while a single woman can have four children at most. This is coherent with the literature on natural fertility such as Tietze (1957) for Hutterite women marrying at age 25, Smith (1960) for the Coco Islands' Malay women, or Henripin (1954) for the first generations in Quebec.

The simulated moments that we obtain are represented in Figures 5 and 6. We fit data for married women better because we have more observations for married than for single women (1,055,171 observations representing 19,882,890 individuals for married women and 71,832 observations representing 1,498,555 individuals for single woman). This gives more weight to married women in the objective function. We are able to reproduce that the difference between the completed fertility of single and married mothers is no greater than one child (Fact 1), and the U-shaped relationship between childlessness and education for both married and single women (Fact 2).

As a test of the theory, we compare our results to three empirical observations that were not used to identify the parameters. These are the marriage rates of men and women and the fertility of husbands. Figure 7 shows that the model reproduces the hump-shaped relationship between marriage rates and education levels (Fact 3). However, the model does not replicate the percentage of marriages for the extreme categories of education among men well. For highly educated men, the proportion of time they have to spend in rearing children (around 40%) is too high, implying that having children and being married is too costly for them. Setting $\alpha = 1$ increases the marriage rates of highly educated men but the marriage rates of highly educated women are now strongly underestimated as they lose their incentive to marry and have children (we discuss this in more detail in Appendix C.3). We

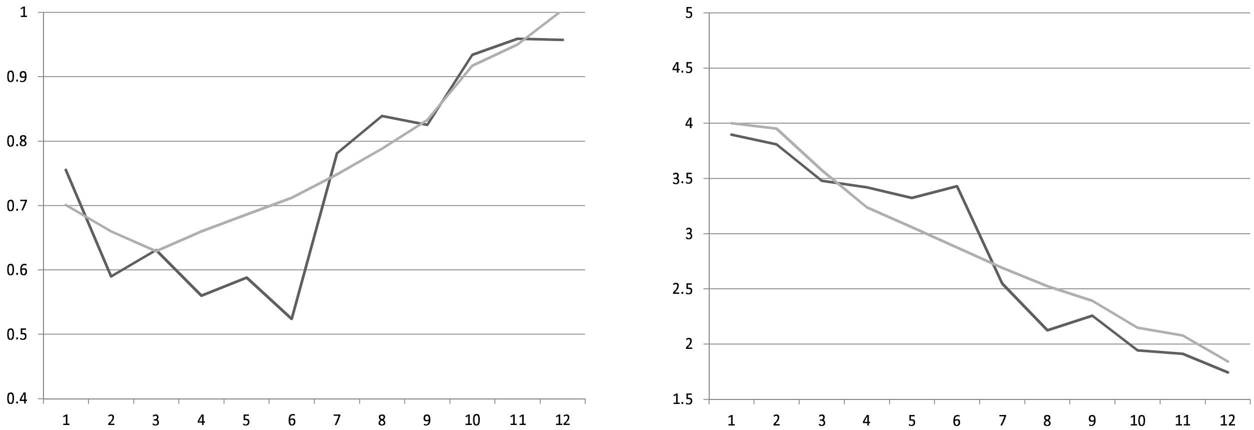

Figure 6: Childlessness Rate and Completed Fertility of Mothers, by Education Categories, Single Women. Data (black) and Simulation (grey)

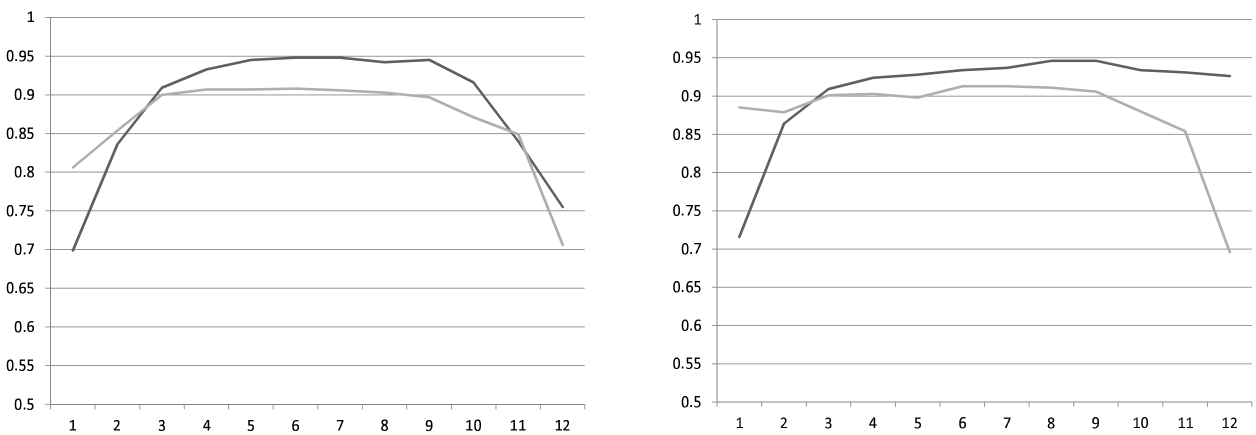

Figure 7: Marriage Rates of Women and Men, by Education Categories. Data (black) and Simulation (grey)

can also reproduce the negative relationship between average fertility and the education of husbands, although we underestimate its slope (see Appendix C.4). In contrast with the marriage rates, in order to better fit husband's fertility rates, we would need a lower value of α (higher opportunity cost for men).

In order to analyze the role $\underline{\theta}$, we ran an estimation under the constraint $\underline{\theta} = 1$ (equal negotiation power of spouses). The cost of marriage drastically increases for a highly educated individual matched with someone with very low education: he/she will have to give up a large part of his/her income to ensure equal consumption in the household. As a result, highly educated will reject unions with lowly educated more often and the percentage of married people in the extreme education categories decreases sharply. As heterogamous unions become less valuable, marriage rates also decrease on average. Assuming an exogenous degree of assortative matching, as in Appendix C.5, greatly helps to recover more realistic marriage rates. Finally, as poor women are less often married, they are also less protected against involuntary childlessness when $\underline{\theta} = 1$. Then the model tends to overestimate the percentage of childlessness among lowly educated singles but the relationship between childlessness and education decreases over a larger range of education categories (from one to six).

Table 3 provides the proportion of women in each regime.[20] We see that uneducated single women are either involuntarily childless (II) or have the maximum fertility (V), depending on their non-labor income (as in Figure 1). Poor married women are either involuntarily childless (VII), in the eat and procreate regime (VIII), or with the maximum number of children (X and XI). This is consistent with Figures 2 to 4. Maximum fertility regimes, where poorly educated women would like to have more children but are constrained by their time endowment, concern 4.3% of American women (0.9% singles and 3.4% married). In Regimes VIII and X, an increase in wages does not increase the consumption of the wife, who consumes c_{m}. It could however increase her fertility (in VIII). We estimate that 1.5% of American women are this situation. This means that although aggregate fertility data suggests that Malthusian checks no longer prevail and Becker's model describes the negative relationship between education and fertility well, our model detects that some categories of the population are still affected by Malthusian mechanisms.

The largest fraction of the population is married with children (77.7% of American women). It is this which allows us to replicate the downward-sloping relationship between fertility and mother's education. Women with the highest education are either voluntarily childless (single in IV or married in IX), or married mothers in the interior regime VI. Regime IX is the regime that we have in mind for DINKs while Regimes VI, VIII, X and XI are the corresponding regimes for DEWKs.

Figure 8 plots the three causes of childlessness as a function of education. Considering all education categories, non-natural involuntary childlessness ("social sterility") concerns 5% of American women (4.8% singles and 0.2% married), while voluntary childlessness concerns 6.6% of American women (3% singles and 3.6% married). We see that childlessness concerns essentially either very poor (involuntarily childless) or very rich women (mostly voluntarily childless), for both married and single people. This is in line with Figures 1 to 4. The percentage of involuntarily childless women decreases with education, while the percentage of voluntarily childless women increases with education. This explains the U-shaped relationship between education and childlessness (Fact 2).

[20] Note that Regime III (eat and procreate for single women) is not present in the simulations. This comes from the discrete choice of fertility in the quantitative analysis which does not provide a consumption level equal to c_{m}.

	single				married						nat.
	I	II	IV	V	VI	VII	VIII	IX	X	XI	steril.
1	0.0	13.0	0.0	6.0	14.3	6.1	39.6	0.0	2.2	15.5	3.3
2	0.2	9.2	0.0	4.9	32.3	4.1	31.2	0.0	0.4	14.2	3.4
3	1.2	5.9	0.0	2.7	67.0	1.2	8.6	0.9	0.0	9.0	3.5
4	1.6	5.6	0.2	1.7	77.9	0.3	1.1	1.8	0.0	6.2	3.4
5	1.7	5.6	0.5	1.3	79.9	0.1	0.2	2.3	0.0	5.0	3.4
6	1.8	5.5	0.8	1.0	80.9	0.0	0.0	2.7	0.0	3.9	3.4
7	1.8	5.4	1.4	0.7	81.1	0.0	0.0	3.2	0.0	3.0	3.3
8	1.7	5.1	2.4	0.5	81.0	0.0	0.0	3.8	0.0	2.2	3.3
9	1.5	4.2	4.2	0.3	80.4	0.0	0.0	4.6	0.0	1.6	3.2
10	1.1	1.4	10.2	0.2	77.0	0.0	0.0	6.4	0.0	0.7	2.9
11	0.9	0.3	14.0	0.1	74.1	0.0	0.0	7.4	0.0	0.4	2.7
12	0.4	0.0	29.3	0.0	58.9	0.0	0.0	9.2	0.0	0.1	2.0
all	1.6	4.8	3.0	0.9	77.7	0.2	1.5	3.6	0.0	3.4	3.3

Table 3: Marital and Fertility Regimes as a Function of Women's Education in %

The hump shaped relationship between marriage and education is related to the high childlessness rates of uneducated and highly educated single women: marrying women who are either not fit to procreate (or would require massive help from their husband to do so) or have low incentives to have children (high opportunity cost) is less attractive.

Despite having assumed pure random matching in the marriage market, the model predicts that individuals are more likely to marry someone with a similar level of education. This level of assortativeness is lower in the simulations than in the data because of the static nature of the model and the assumption that life only brings one chance to get married to a random person. In Appendix C.5, we provide a way to measure the degree of assortativeness in the data and the model. The model accounts for 24% of the variation in the assortativeness of matching. In reality, the assortativeness is higher because, first, people meet several possible matches (we would need a dynamic model to reproduce this), and second, individuals are more likely to meet others who have similar levels of education to their own.

With a model able to match the level of assortative matching, in particular among poor people, we expect a lower c_{m}. In fact, 53% of women with no education marry a man with no education. This is much higher than in our model. Consequently, our estimated value of c_{m} is probably too high because we have more women with no education marrying men with higher levels of education than in reality. This means that in order to have

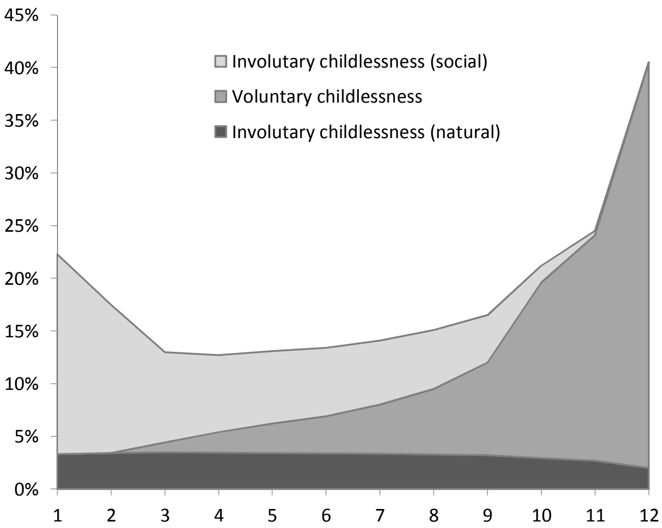

Figure 8: Childlessness Causes, per Education Level (simulations)

involuntary childlessness, the constraint on c_{m} must be higher. A way to increase the degree of endogenous assortativeness consists in modifying the bargaining rule (6) by making it less sensitive to the wage ratio. This will increase the rejection rate of matches with very different people. However, achieving more assortativeness will be at the expense of matching a reasonable marriage rate, as people only meet once in our static set-up. This is a limitation of our approach.

A complementary and exogenous way to generate the right degree of assortativeness is to assume, as in Fernández-Villaverde et al. (2010), that a fraction λ of the female population draws a possible match from her education category, while $1 - \lambda$ draws from the total population. For each chosen value of λ, we can estimate the remaining parameters and see whether changes are important.[21] We provide the results in Appendix C.5, Table 16. Our main results are robust to this change. For instance, when λ equals 0.3, the fit of the model does not change significantly although the marriage matrix becomes much more satisfying.

Altogether, we can conclude that the benchmark model allows to understand the three facts of the introduction for the year 1990.

[21] In Fernández-Villaverde et al. (2010), a fraction of the population is randomly matched, the rest being matched following a Gale-Shapley algorithm, which generates perfect assortativeness.

5 Counterfactual experiments

We have seen that the model characterizes well the relationship between childlessness and fertility and the education of women, over a cross-section of individuals. Now, we use these identified relationships in several counterfactual experiments. The first is a historical experiment to understand the changes in marriage rates, childlessness and fertility over the periods 1960-1990 and 1990-2010. Then, we also look at how an increase in inequality and a change in the wage gap affect marriage rates, childlessness and fertility.

5.1 Rise in Education from 1960 to 1990

The number of people who completed Grade 12 (category 7) increased from 1960 to 1990 while those with an education lower than Grade 11 were fewer in 1990 than in 1960. This implies an overall increase in the education level of both men and women.

5.1.1 Rise in Men's Education

Let us first consider the effect of the rise in men's education. Keeping the identified parameters constant, we simulate the moments using the proportions of men in each category of education for the year 1960 and compare them with the simulated moments using the proportions for 1990.[22] We then compare the change in the simulated moments with the change in actual data over the period 1960-1990.

The left panel of Figure 9 shows that the empirical marriage rates decreased for low skilled and increased for high skilled women between 1960 and 1990. This qualitative change is well accounted for by the model, as shown in the right panel of Figure 9. This implies that the rise in men's education explains the shift in the relationship between marriage and education level well. Women with low education marry less often in 1990 because they are more often turned down by more educated men while highly educated women are more likely to accept an offer.

In 1960, completed fertility for single women was not given in the Census. We are thus limited to analyzing changes in childlessness and fertility for married women only. In Figure 10,[23]

[22] The proportions of men and women in each education category in 1960, 1990 and 2010 are shown in Appendix A.8.

[23] The relationship between childlessness and education in 1960 was not U-shaped. In Subsection 5.3 we argue that a possible reason for this is increasing inequality.

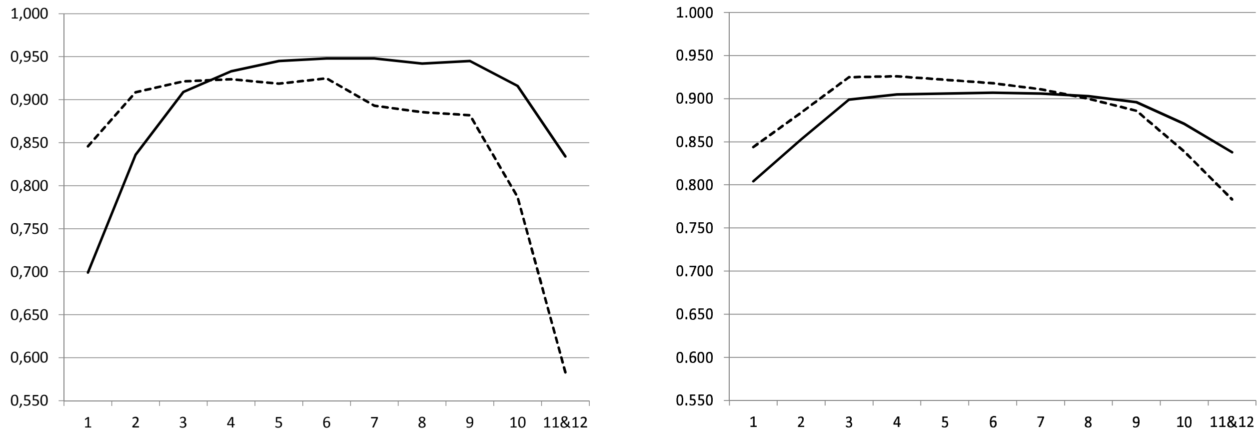

Figure 9: Marriage Rates, Data (left) and Simulation (right), 1960 (dots) and 1990 (solid)

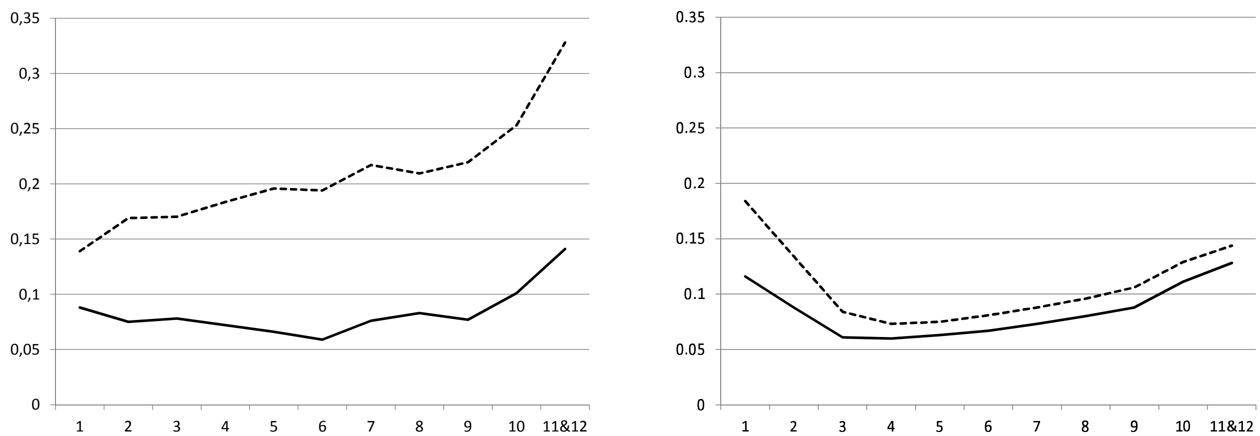

Figure 10: Childlessness Rates, Data (left) and Simulation (right), 1960 (dots) and 1990 (solid)

the left panel shows that the childlessness rates of married women decreased over this period and the right panel shows that our model is partly able to explain this. Quantitatively, our model accounts for a third of the drop. Women with low education marry richer men, which allows them to escape the minimum consumption constraint.[24]

The fertility of married women increased over the period 1960-1990, for all education categories, as shown in the left panel of Figure 11. The rise in men's education, on the contrary, predicts a drop in fertility rates, due to higher opportunity cost of rearing children for more educated men. The right panel of Figure 11 shows that this drop is very slight. The failure to reproduce the rise in fertility is not surprising given that this period corresponds to the exceptional event of the baby boom. Since childlessness rates decreased and the fertility of mothers increased, total fertility increased during this period. The simulations are not able

[24]To generate a larger drop, a model where the fraction of childrearing supported by women α is covered by baby sitters would be needed (see Hazan and Zoabi (2012)).

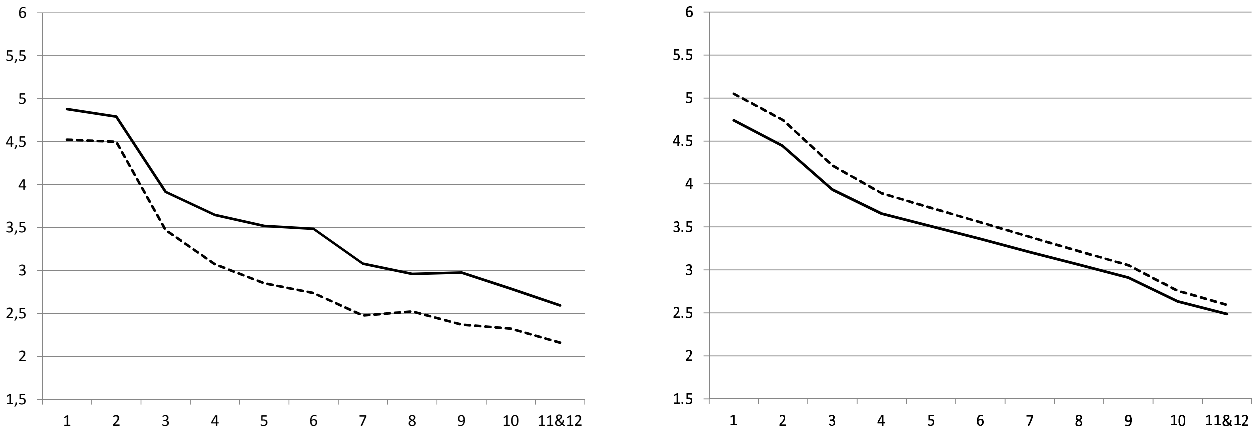

Figure 11: Completed Fertility Rates, Data (left) and Simulation (right), 1960 (dots) and 1990 (solid)

to reproduce this, even though we can reproduce some of the decrease in childlessness.

In the theory, an increase in men's education reduces both childlessness rates and the fertility of mothers. The fact that these two variables move in the same direction shows that they are affected by different mechanisms. Childlessness decreases because women marry richer men and this decreases involuntary and voluntary childlessness. The fertility of mothers decreases because fathers face a higher opportunity cost.

5.1.2 Rise in Women's Education

To evaluate the effect of the rise in women's education, we computed aggregate indicators of fertility using the education shares of 1960 and 1990. Table 4 decomposes the change in fertility into two parts: the share effect and the shift effect. The share effect captures the impact of the increase in the education of women. It is given by the comparison between the aggregate fertility in 1960 and the aggregate fertility in 1960 computed with women's education shares in 1990 (men's education shares are fixed to those in 1960). Both data and simulation indicate a drop of the same magnitude, related to the overall increase in the education level of women, who have fewer children because their opportunity cost is higher. The shift effect corresponds to the shift in the curve of Figure 11 (right) and is obtained by comparing the aggregate fertility in 1990 with the fertility in 1960 if women's education shares had been those in 1990. This effect is positive in the data, but negative in the simulation, as explained in the previous subsection.

We are not able to reproduce the increase in mothers' completed fertility because some women in the 1990 sub-sample are the mothers of the baby boom generations. Explaining

	1960	1960 (shares 1990)	1990	Shares	Shift	Total
	(1)	(2)	(3)	(1)-(2)	(2)-(3)	(1)-(3)
Data	2.525	2.052	2.902	-0.472	0.850	0.378
Simulation	3.495	3.053	2.947	-0.442	-0.106	-0.548

Table 4: Decomposition of the Change in Aggregate Fertility

this goes beyond the scope of this paper. However, this historical experiment tells us that without the baby boom, childlessness would have decreased by less and the completed fertility of mothers would not have increased. The difference in the shift effect between simulation and data constitutes a measure of the impact of the baby boom on aggregate fertility: $0.106 + 0.850 = 0.956$ children per married woman.

5.2 Rise in Education from 1990 to 2010

From 1990 to 2010, education attainments have further increased (Appendix A.8). This coincides with a significant decrease in marriage rates from 93% to 85.9%. In particular, marriage rates decreased for all education categories except for PhDs, as shown in Figure 12 (left). In this subsection, we compute the impact of the variation in educational achievement on the decrease in marriage rates.

Using the shares provided in Table 10 for the year 2010 and keeping the identified parameters for 1990 fixed, we can compute the marriage rates predicted by the model. We then compare the variation between the theoretical marriage rates to the data for the period 1990-2010. As shown in the right panel of Figure 12, the model predicts a general decrease of marriage rates for women (the pattern is similar for men) from 91.4% to 88.5%. This corresponds to 41.2% of the observed decrease in the marriage rates. Hence, changes in education explain almost half of the dramatic change in marriage rates. The remaining part of the drop may be explained by the demise of strict monogamy (see de la Croix and Mariani (2012)), which reduces the number of "ever married" persons.

The model is also able to reproduce qualitatively the increase in the marriage rate for PhD women. The main reason for this lies in the ex-post assortative matching that takes place. As the proportion of men with a PhD increases, highly educated women have a higher chance to be matched with a highly educated man and consequently, to agree to get married. We remain short on this increase mainly because of the absence from the model of *ex-ante* assortative matching on the marriage market.

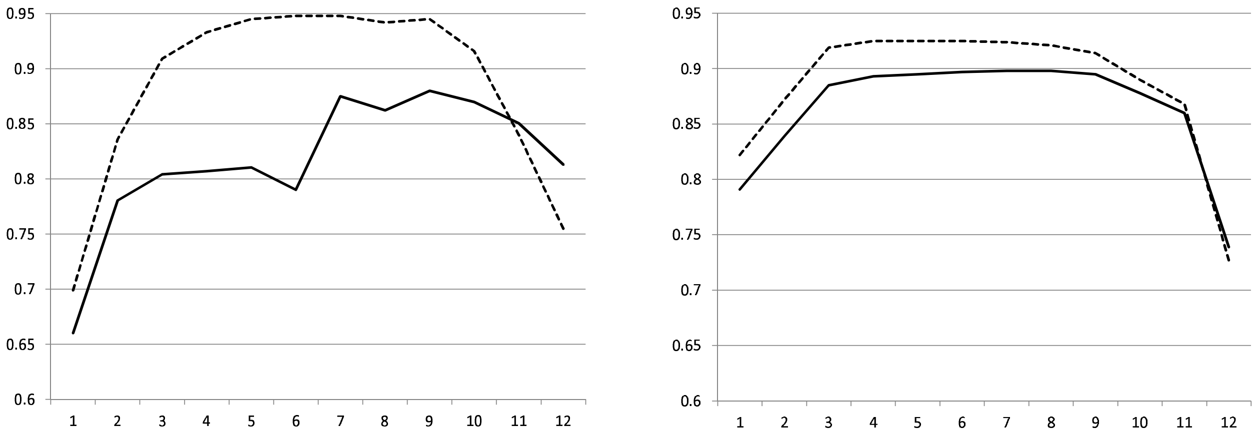

Figure 12: Marriage rates, women, data (left) and simulation (right), 1990 (dots) and 2010 (solid)

5.3 Increase in Inequality

Since 1980, income inequality and skill premia have been on the rise in the U.S. To assess its effect on family patterns, we implement a rise in inequality through a change in the Mincer parameter ρ of Equation (8). We increase the return of one additional year of education from 10% to 12%, keeping the wage of the largest category constant (Category 7); the change in the Mincer parameter thus increases the spread around the wage of high school graduates. To fix ideas, this corresponds to an increase in the Gini coefficient computed on $w^f + a^f$ from 0.166 to 0.174.

From our theory, we anticipate that inequality increases both types of childlessness. As the poor become poorer, the minimum consumption constraint binds more often and involuntary childlessness rises. For the rich, the opportunity cost of having children is increased by the higher skill premium (for both men and women), and more of them choose to remain childless. We obtain that U-shaped pattern is clearly more pronounced for the higher Mincer coefficient, and the effect on poorly educated women is particularly strong (see the figure in Appendix C.6). On the whole, the number of women in involuntary childlessness regimes II and VII increases by 20% (from 5% of the population to 5.9%). Going back to Figure 10, where the U-shaped pattern emerged between 1960 and 1990, we can conclude that the rise in wage inequality would be one of its driving forces.

The rise in inequality also affects the marriage rates of the highly educated. As women in education categories 9 to 12 get richer, they become more demanding, and marry less often. Hence, a rise in inequality is another candidate to explain the drop in marriage rates observed from 1990 to 2010 (Section 5.2). Finally, the rise in inequality increases the slope of the relation between the fertility of mothers and education as the opportunity cost of poorer

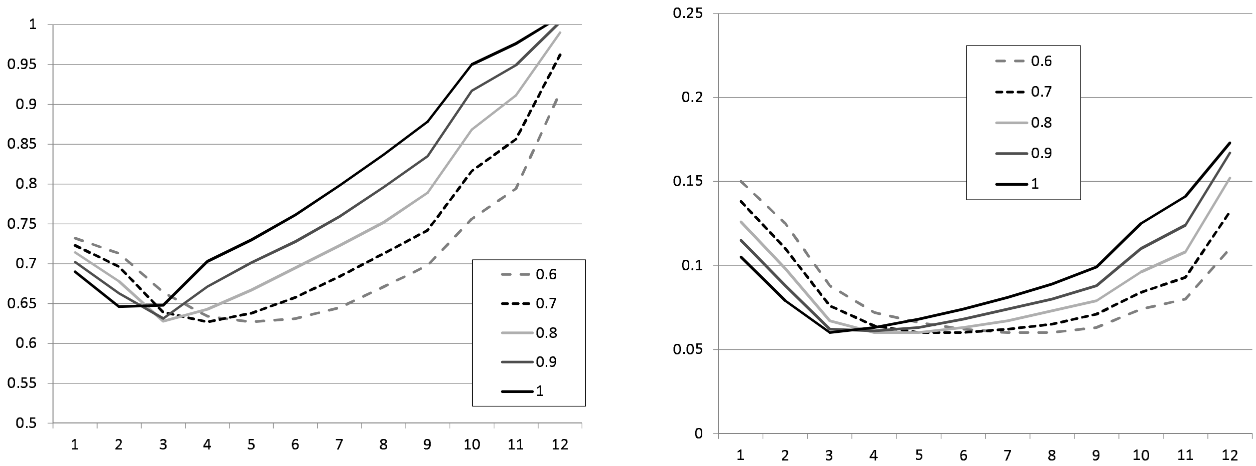

Figure 13: Childlessness Rate for Different Wage Gaps - Single (left) and Married (right)

women decreases while it increases for the richer ones.

5.4 Changes in the Gender Wage Gap, γ

Another first order change of the last decades is the closing of the gender wage gap (Goldin (1990) or Jones et al. (2003)). To assess its effect on family patterns, we simulate the model for various values of the parameter γ of Equation (8), keeping the other parameters fixed.

Figure 13 shows the effect on childlessness for single (left) and for married (right) women. As predicted by the proposition in Appendix B.1, there will be fewer poor women in the involuntary childlessness regime, but educated women are more likely to be voluntarily childless (left panel). The same applies for married women. Closing the gender wage gap is therefore a powerful tool to fight the involuntary component of childlessness.

A change in the wage gap has the usual properties on the aggregated fertility of mothers: a decrease in the wage gap decreases fertility since it increases the opportunity cost of rearing children. This is however not true for single women in the corner regime with maximum fertility for whom fertility does not depend on wage: the effect of a higher relative wage is not large enough for them to exit this regime. To get an idea of the magnitude of the effect of wage w^f on fertility, we compute the elasticity of total fertility to the wage gap for the largest group (education category 7, married women). When the wage gap closes from 0.8 to 0.9, fertility drops from 3.40 to 3.22, which gives an elasticity of -0.05.

The wage gap also has an effect on marriage rates. Poor women will marry more often, while highly educated women will marry less often. This effect is particularly large for Ph.Ds: if the wage gap goes from 0.8 to 1, their marriage rate drops from 0.779 to 0.624.

6 Conclusion

To analyze fertility behavior we distinguished explicitly the decision to have children from the choice of the number of children. This distinction turned out to be important, both in terms of data and theory.

Data shows the following three facts. First, single women are much more likely to be childless but, when mothers, their fertility is lower by no more than one child than that of married mothers. Second, childlessness exhibits a U-shaped relationship with education for both single and married women. Third, there is a hump-shaped relationship between marriage rates and education levels. These facts are robust for different races and age cohorts.

We have developed a model that allows us to analyze the effect of men's and women's incomes on fertility going beyond the usual distinction between income and substitution effects. Both non-labor income and wages play a complex role, shaping the incentives to agree to marry or not, and affecting the allocation of resources in the couple.

The main conclusion from the theory is to identify several "regimes" and the conditions under which these regimes will prevail. Some of these regimes are new compared to the literature, and turn out to be quantitatively important. Involuntary childlessness can have natural or social causes. Social involuntary childlessness regimes appear for women with low education and low non-labor income, either single or married; we estimate that they account for 5% of American women. In the "eat and procreate" regime, the income of the woman is not high enough for her to be fit to procreate, but it is optimal for her husband to abandon part of his consumption in order to be able to produce children within the couple. This should be highlighted: although aggregate fertility data suggests that Malthusian checks do not prevail any more and Becker's model describes the relationship between education and fertility well, our model detects that some categories of the population are still affected by Malthusian mechanisms. In the voluntary childlessness regime, highly educated women do not have children because of their high opportunity cost.

Our theory also provides a framework to interpret childlessness for both single and married women, allowing for involuntary childlessness for uneducated women and voluntary childlessness for highly educated ones. Simulations show that those regimes are not "empty", and concern a significant fraction of the population. The relatively high percentage of the population in these regimes allows us to understand the U-shaped relationship between education and childlessness highlighted in the stylized facts. Still, the majority of the population is married, in the interior regimes, which allows us to replicate the usual downward sloping relation between fertility and mother's education.

Marriage interacts with childlessness in two ways; for poor women, marriage is an opportunity to get enough resources to be able to have children. Hence, marriage reduces involuntary childlessness. For rich women, marriage reduces the opportunity cost of having children, as husbands also help with raising the children; it therefore also reduces voluntary childlessness.

Identifying the structural parameters of the model using a simulated method of moments technique shows that the features of the data on fertility and childlessness are well captured. On the whole, our model provides a way of understanding the relationship between fertility and education, childlessness and education, and finally, marriage and education all taken together.

Using the model to understand the changes that occurred over the period 1960-1990 we have learned that an increase in the education of men leads to a decrease in both involuntary and voluntary childlessness and an increase in the marriage rate of educated people. As the model fits the relationship between mother's education and fertility very well, it also accounts well for the effect of the increase in the average education of women on fertility.

Involuntary childlessness is of particular relevance as far as social welfare is concerned, as it restricts the capabilities of individuals. We have shown that closing the gender wage gap is a powerful tool for limiting the proportion of involuntary childlessness generated by poverty.

A more detailed study in the allocation of time between men and women could give further insights and predictions for changes in today's fertility trends. Allowing for a more complex marriage market structure would also improve the predictions of the model in terms of assortative matching.

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A Data - For Online Publication

A.1 Education Categories

We divide the population into 12 categories of education and report the average number of years of education, e, for each category as well as the number of observations (sum of singles and ever-married) per category. Each of these observations has a weight given by the Census and represents between 2 and 186 individuals.

Nb	Category	e	N. obs.	Nb	Category	e	N. obs.
1	No school	0	12,122	7	Grade 12	12	479,703
2	Grade 1-4	3	14,050	8	1 year of college	13	178,274
3	Grade 5-8	7	84,243	9	2 years of college	14	53,428
4	Grade 9	9	38,121	10	Bachelor degree	16	99,046
5	Grade 10	10	57,213	11	Master degree	17	56,855
6	Grade 11	11	49,413	12	Doctoral degree	20	4,612
				1 to 12			1,127,080

Table 5: Education Categories.

A.2 Cohabitation and Uneducated Single Mothers

When dealing with never-married individuals we might be skeptical about whether these individuals are single or just unmarried with a partner (specially poor women who have many children). In the Census 1990, among never-married women aged 45-70, only 1.8% declared themselves as being with a partner. The percentages for mothers and childless were respectively 3.7% and 1.3%. The percentages vary, however, for different education levels: Table 6 provides the proportion of single mothers, aged 45-70, saying they are with an unmarried partner for each education level.

This table confirms that very few women who have not married are living with a partner. The highest percentages of cohabitation are seen for women aged 36-40, having achieved Grade 1-4, or with a doctoral degree: respectively 21.5% and 21.7% of never-married women in these education levels claim to be unmarried but have a partner.

Bramlett and Mosher (2002) estimate that, in 1995, 42.7% of never married women between ages 40-44 had ever cohabited. Their definition for cohabitation was being unmarried but having a sexual relationship while sharing the same usual address. They also say that the probability of transition to marriage after 1 year of cohabitation is 30%, after 5 years, 70%, and after 10 years, 84%. Consequently, if cohabitation lasts, marriage is very likely to follow.

Rindfuss and VandenHeuvel (1990) compared cohabitants to both married and single individuals and concluded that cohabitants' attitudes were in many respects closer to those of single than of married people. In particular, cohabitants' fertility expectations are more similar to those of single than married people. Indeed, comparing the percentage of childless individuals who expect children, cohabitants are much closer to never-married than to married respondents (among women, 11% of cohabiting, 4% of single and 40% of married people expect to have a child within two years). Consequently, although cohabitation has many of the characteristics of a marriage (sharing a dwelling unit), cohabitants also share some of the characteristics of single people (fertility expectations). This puts cohabitants in a middle position between single and married people.

A second question raised by our facts concerns the identity of those uneducated mothers who have many children. Table 7 gives some information. The column $(1-\phi n)w^f + a^f$ reports the total income of the person. Earnings are in column $(1-\phi n)w^f$. Not surprisingly, earnings decrease with the number of children, probably because hours worked drop as the number of children rises. More interestingly, the column a^f reports the difference between total income and earnings. We observe that fertility increases with a^f, which is a prediction of our model. Now, who are these women? Very few of them report to be with an unmarried partner (see Table 6). Less than half of them are black. A majority head of their household.

Category	%	Category	%
1	3.8	7	3.6
2	4.8	8	2.8
3	4.0	9	3.8
4	3.6	10	3.0
5	3.8	11	2.8
6	4.0	12	8.2

Table 6: Percentage of Single Mothers, Aged 45-70, with an Unmarried Partner

n	nobs.	$(1-\phi n)w^f + a^f$	$(1-\phi n)w^f$	a^f	% with partner	% black	% head of household
1	17,531	7,302.87	5,014.92	2,287.95	6.7%	33.5%	35.0%
2	11,797	7,889.39	5,817.37	2,072.03	9.5%	39.1%	45.3%
3	7,108	6,923.74	4,553.75	2,369.99	11.1%	39.4%	52.5%
4	4,636	6,712.25	3,998.48	2,713.77	6.4%	32.1%	56.8%
5	2,882	6,239.59	3,556.02	2,683.57	5.0%	41.8%	61.1%
6+	5,446	5,844.52	2,576.98	3,267.54	6.1%	36.4%	64.5%

Table 7: Single Mothers with no Education

A.3 NSFG surveys

The National Survey of Family Growth asks women between 15 and 44 years of age questions about their fecundity status and fertility intentions. This allows us to classify each respondent as either voluntarily or involuntarily childless. Looking at the NSFG for the years 1973 and 1976, we have examined in detail 306 childless women between 36 and 44 years old. These women belong to the cohort considered for our stylized facts in Section 2.

We denote a woman as voluntarily childless if: (1) she was voluntarily sterilized for contraceptive reasons, (2) she has always used contraception, (3) she has been pregnant and aborted, (4) she does not intend to have children but does not report any difficulty in becoming pregnant, or (5) she does not intend to have children but does not report any difficulty conceiving or delivering a baby for her or her husband.

We denote a woman as involuntarily childless if she would like to have children and one of the following conditions is met: (1) she has not been using birth controls for at least two years but has become pregnant, (2) she has never used contraception and has been married for a long period, (3) she reports either a difficulty or the impossibility of having a baby, or (4) she has problems or difficulties to conceiving or delivering a baby for her or her husband (some women had had up to six miscarriages).

We assume that a woman wants a child if she says that (1) she wants to become pregnant as soon as possible, (2) she would like a baby when pregnancy is dangerous or impossible, (3) she plans to adopt if she cannot have a child of her own, (4) she has adopted and does not use contraception because she does not mind getting pregnant, (5) she wanted children before marriage and never used contraception, or (6) she talked with her doctor about increasing her chances of having a baby.

We did not know how to classify some women, either because information was missing or the

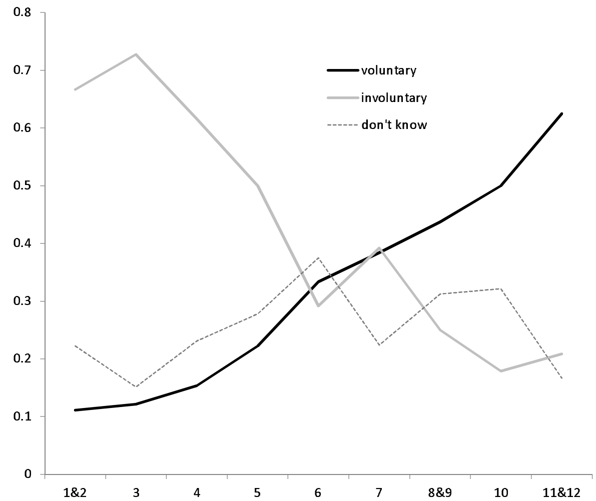

Figure 14: Involuntary and Voluntary Childlessness (in %), by Education Category

information was contradictory. Here are some representative examples of these women: (1) she has never used contraception but says that she decided not to have children and has no problem becoming pregnant, (2) she used contraception, reports being able to procreate and is seeking pregnancy or intends to have a child in the next two years (she could become a mother soon), or (3) she or her husband remained sterile not for contraceptive reasons (but, for example, through accident or illness), but were still young enough to procreate.

Figure 14 shows the relationship between involuntary and voluntary childlessness and the level of education. Notice that the U-shaped relationship between childlessness and education also holds in this dataset.

A.4 Five-year cohorts

The stylized facts highlighted in Section 2 can also be found if we consider each 5-years cohort separately. Figures 15 and 16 show the relationship between childlessness and education and the fertility of mothers and education for married and single women for each cohort. The only major difference between cohorts is in the childlessness rate of single women, with intermediate levels of education (Figure 16): older single women were much less likely to be mothers than younger single women.

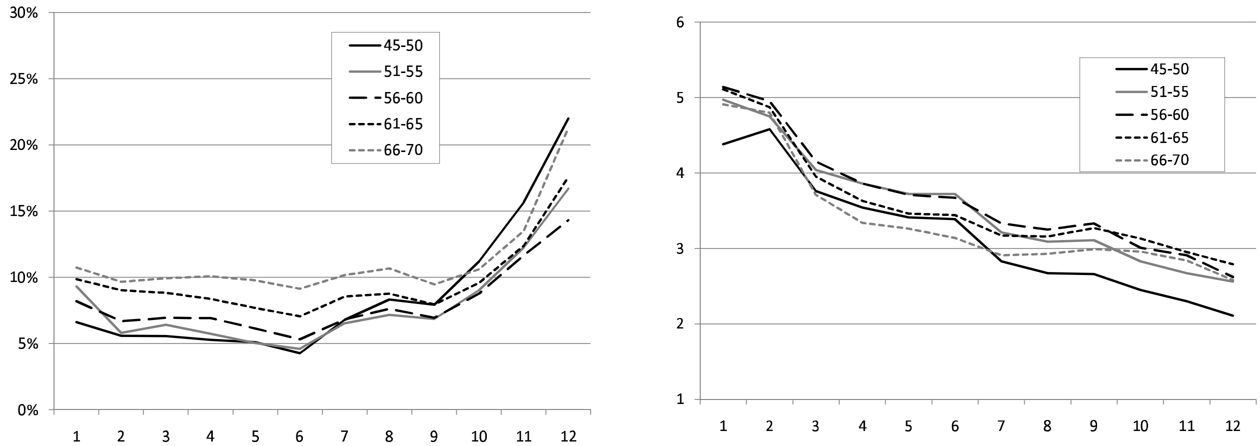

Figure 15: Childlessness Rate and Completed Fertility of Mothers, by Education Category, Married Women

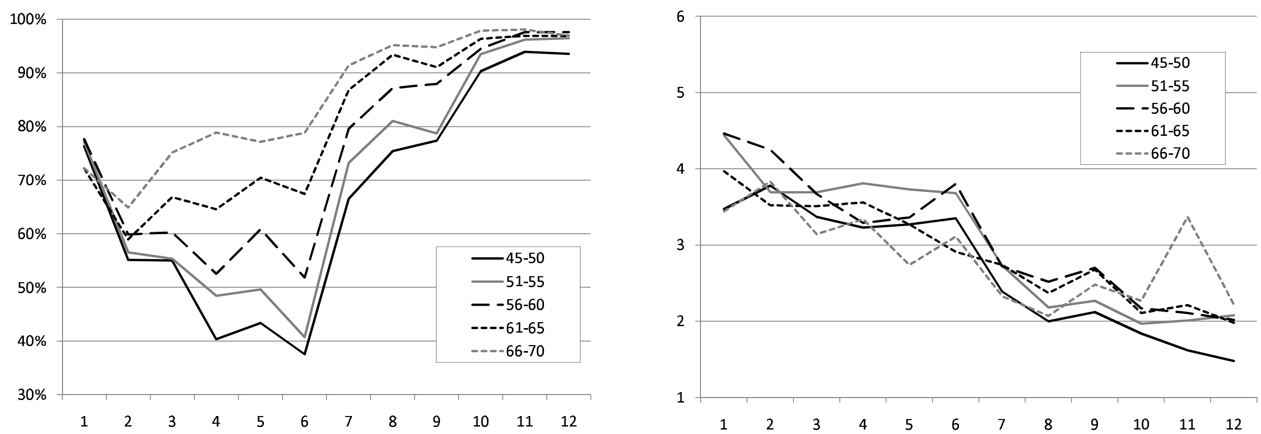

Figure 16: Childlessness Rate and Completed Fertility of Mothers, by Education Category, Single Women

A.5 Standard errors of the mean

Table 8 reports the standard errors of the mean for the completed fertility and the childlessness rates of both single and married mothers.

Education Category	Childlessness Rates ($\times 10^{-3}$)		Completed fertility of Mothers	
	Married	Single	Married ($\times 10^{-3}$)	Single ($\times 10^{-2}$)
1	0.70	1.61	7.49	2.205
2	0.55	2.35	6.30	2.154
3	0.23	1.29	2.06	1.158
4	0.32	2.27	2.62	1.676
5	0.25	2.03	1.98	1.569
6	0.25	2.29	2.09	1.626
7	0.09	0.60	0.57	0.621
8	0.15	0.83	0.86	0.938
9	0.27	1.60	1.55	1.615
10	0.23	0.62	1.07	1.556
11	0.35	0.47	1.41	2.201
12	1.47	1.34	5.08	4.875

Table 8: Standard Errors from U.S. Census 1990

A.6 Races

In this appendix, we split the population into five racial groups, Whites, Blacks, Natives, Asians and Hispanics. We constructed each race group from two variables: RACE and HISPAN. In order for an individual to be considered as White, Black, Native or Asian, he or she has to be from that particular race and "not Hispanic". We do not know the race of 435 observations so the sum of the races is not equal to the sum of the observations. Table 9 gives the number of observations (unweighted) by education category for each race. Figures 17 and 18 show the childlessness rate and completed fertility of mothers, by education category, for married women and single women respectively.

For all groups, we find a negative relationship between the fertility of married mothers and education. For single mothers, we have few observations for Asians and Natives so the relationship is not as clear. For other single mothers, the relationship holds for Blacks and Hispanics and is slightly hump-shaped for Whites. The fertility differential between single and married mothers is larger for white women than for others: married mothers have between 1.5 and 2 times more children than single mothers. Hispanic married mothers have

	Blacks	Whites	Natives	Asians	Hispanics
1	1,264	4,631	274	1,518	4,400
2	1,673	5,276	200	868	6,002
3	8,897	59,603	806	2,397	12,487
4	4,029	29,879	300	832	3,059
5	5,468	47,968	471	680	2,598
6	5,877	40,798	361	338	2,018
7	24,421	429,963	1,985	8,115	15,078
8	8,787	160,406	937	,2988	5,104
9	2,631	46,909	275	1,758	1,838
10	4,092	88,089	262	4,551	2,025
11	3,641	49,819	164	1,941	1,283
12	242	3,974	16	207	172
Total	71,022	967,315	6,051	26,193	56,064

Table 9: Number of Observations by Education Category and by Race

no more than one child more than single mothers for all education categories. For Blacks, the difference between the fertility of married and single mothers decreases as education increases. Both the U-shaped relationship between childlessness and education and the hump-shaped relationship between marriage rates and education hold in general. In particular for Asian married women, childlessness always increases with education; childlessness of single Hispanic women is flat for the first education levels and then increases. Comparing across groups, we see that Black women are in general more likely to be childless if married, less likely to be childless if single and that the U-shaped relationship (Fact 2) is more pro-

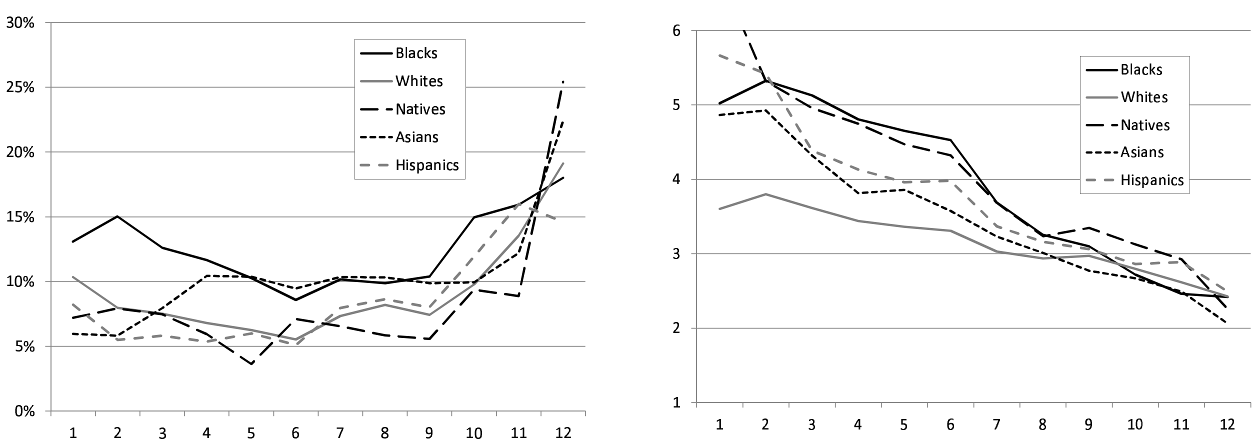

Figure 17: Childlessness Rate and Completed Fertility of Mothers, by Education Category, Married Women

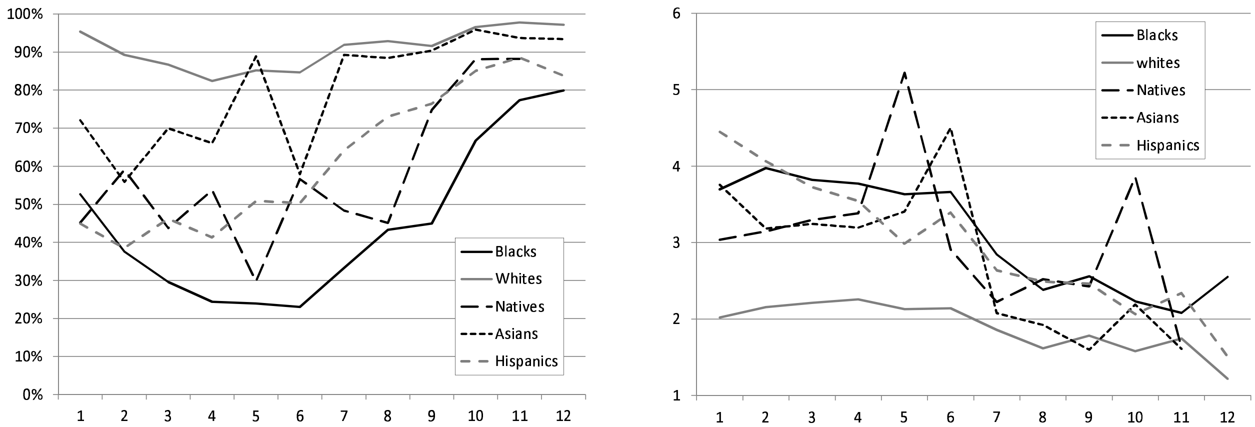

Figure 18: Childlessness Rate and Completed Fertility of Mothers, by Education Category, Single Women

nounced for them than for other groups. Both single and married Black mothers have more children than White mothers. Differential fertility between groups decreases with education.

A.7 Disability

The U-shaped relationship between childlessness and education is highly affected when we consider the data without individuals who have any lasting physical or mental health condition that prevents or causes difficulty working, living independently or taking care of their own personal needs (respectively, variables DISABWRK, DIFFMOB and DIFFCARE of IPUMS). Note that these variables say nothing about the ability to reproduce. For married women the relationship between childlessness and education is not affected, but the relationship for single women becomes flat for the first education levels and only increases from Grade 11 on.

Our position is that disabled women are, *de facto*, lowly educated and the constraint $c^f > c_{\mathrm{m}}$ reflects their incapacity to have children when they do not have a husband investing in them. Moreover, in 1990 the Census did not distinguish birth defects from disabilities acquired later in life.[25] Poor working conditions for the lowly educated is likely to affect their health aged 45-70. There is then a clear endogenous problem in the relationship between disability and education: adults who lived in the worst conditions (the ones with the lowest education) are the most likely to suffer health problems when older. In other words, the poor are more likely to have health problems when old than the rich. Consequently, we argue in favor of keeping the disabled in the dataset.

[25]See http://www.census.gov/hhes/www/disability/sipp/disab02/ds02t2.html for information about disability types and their proportions in the U.S.

A.8 Education Shares

Education level	1960		1990		2010	
	men	women	men	women	men	women
1	0.024	0.019	0.012	0.011	0.015	0.014
2	0.085	0.057	0.018	0.013	0.009	0.009
3	0.427	0.393	0.094	0.072	0.036	0.031
4	0.064	0.068	0.036	0.033	0.016	0.013
5	0.070	0.078	0.047	0.050	0.022	0.019
6	0.046	0.048	0.035	0.043	0.023	0.020
7	0.149	0.203	0.316	0.422	0.287	0.309
8	0.027	0.033	0.168	0.161	0.159	0.159
9	0.032	0.040	0.042	0.048	0.080	0.099
10	0.040	0.039	0.125	0.090	0.206	0.201
11 & 12	0.036	0.021	0.107	0.057		
11			0.089	0.052	0.127	0.117
12			0.018	0.005	0.023	0.011

Note: Education categories 11 and 12 were merged in the 1960 Census. Data for 2010 is taken from the American Community Survey (ACS).

Table 10: Proportions of Individuals in Each Education Category

B Theory

B.1 Optimal Decisions of Single Women

In this appendix, we first provide a definition of non-labor income and wage thresholds useful to establish a complete description of the choices made by single women in function of their wage and their non-labor income. We then provide Proposition 2 and prove it.

Definition 1 (Non-labor income thresholds for singles)

$$\underline{a} = c_m \left(\frac{\phi(\nu + \eta) - 1}{\phi \nu} \right) + \mu \quad , \quad \overline{a} = c_m + \mu > \underline{a}$$

Definition 2 (Wage thresholds for singles)

$$\mathcal{W}_0^f(a^f) = \frac{c_m + \mu - a^f}{1 - \phi\eta}, \quad \mathcal{W}_2^f(a^f) = \frac{2c_m + \mu - a^f}{1 + \phi(\nu - \eta)},$$
$$\mathcal{W}_3^f(a^f) = \frac{a^f - \mu}{1 + \phi(\nu - \eta)}, \quad \mathcal{W}_5^f(a^f) = \frac{a^f - \mu}{\phi(\nu + \eta) - 1},$$

$\mathcal{W}_1^f(a^f)$ is the smallest root of the quadratic equation in w^f:

$$\left(w^f + a^f - \mu\right)\nu = c_m \left(\frac{w^f(1 - \phi\eta) + a^f - \mu - c_m}{\phi w^f} + \nu\right), \tag{9}$$

$\mathcal{W}_4^f(a^f)$ is the smallest root of the quadratic equation in w^f:

$$\ln\left(w^f + a^f - \mu\right)\nu = 2\ln\frac{w^f(1 + \phi(\nu - \eta)) + a^f - \mu}{2} - \ln\left(\phi w^f\right).$$

Proposition 2 (Fertility of singles) *Under Assumption 1, the optimal choice of a single woman with non-labor income a^f and wage w^f is given by:*

1. *when $a^f < \underline{a}$:*
   - $\forall w^f < \mathcal{W}_0^f(a^f)$, $c^f = c^f_{II}$, $n = 0$,
   - $\forall w^f \geq \mathcal{W}_0^f(a^f)$, $c^f = c^f_{IV}$, $n = 0$.
2. *when $a^f \in [\underline{a}, \overline{a}[$:*
   - $\forall w^f < \mathcal{W}_0^f(a^f)$, $c^f = c^f_{II}$, $n = 0$,
   - $\forall w^f \in [\mathcal{W}_0^f(a^f), \mathcal{W}_1^f(a^f)[$, $c^f = c^f_{IV}$, $n = 0$,
   - $\forall w^f \in [\mathcal{W}_1^f(a^f), \mathcal{W}_2^f(a^f)[$, $c^f = c_m$, $n = n_{III}$,
   - $\forall w^f \in [\mathcal{W}_2^f(a^f), \mathcal{W}_4^f(a^f)[$, $c^f = c^f_{I}$, $n = n_{I}$,
   - $\forall w^f \geq \mathcal{W}_4^f(a^f)$, $c^f = c^f_{IV}$, $n = 0$.
3. *when $a^f \geq \overline{a}$:*
   - $\forall w^f < \mathcal{W}_3^f(a^f)$, $c^f = c^f_{V}$, $n = \bar{n}_M$,
   - $\forall w^f \in [\mathcal{W}_3^f(a^f), \mathcal{W}_4^f(a^f)[$, $c^f = c^f_{I}$, $n = n_{I}$,
   - $\forall w^f \geq \mathcal{W}_4^f(a^f)$, $c^f = c^f_{IV}$, $n = 0$.

Proof. To prove Proposition 2, we need to show that under Assumption 1:

1. when $a^f < \underline{a}$, $\mathcal{W}_5^f(a^f) < \mathcal{W}_2^f(a^f) < \mathcal{W}_0^f(a^f)$ and $\mathcal{W}_0^f(a^f)$ is greater than the highest root of Equation (9);

2. when $a^f \in [\underline{a}, \overline{a}[$, $\mathcal{W}_0^f(a^f) < \mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$;

3. when $a^f \geq \overline{a}$, $\mathcal{W}_3^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$.

Proof and Implications of 1.

Straightforward computations indicate that when $a^f < \underline{a}$, $\mathcal{W}_5^f(a^f) < \mathcal{W}_2^f(a^f) < \mathcal{W}_0^f(a^f)$. When $w^f \leq \mathcal{W}_0^f(a^f)$, becoming a mother is not feasible either in the interior regime or in the get fit and procreate one.

Once $w^f > \mathcal{W}_0^f(a^f)$, a single woman can have children in Regime III. Comparing utility in Regime IV with utility in Regime III, $u(c_{\mathrm{IV}}^f, 0) \leq u(c_{\mathrm{m}}, n_{\mathrm{III}})$ if and only if,

$$\left(w^f + a^f - \mu\right)\nu \leq c_{\mathrm{m}}\left(\frac{w^f(1-\phi\eta) + a^f - \mu - c_{\mathrm{m}}}{\phi w^f} + \nu\right). \tag{10}$$

The LHS is linear and increasing in w^f and the RHS is increasing and concave in w^f (since $a^f - \mu < c_{\mathrm{m}}$), so Equation (10) holding with equality (i.e. Equation (9)) has at most two solutions. At $\mathcal{W}_0^f(a^f)$, the LHS of Equation (10) is higher than the RHS, implying that $u(c_{\mathrm{IV}}^f, 0) > u(c_{\mathrm{m}}, n_{\mathrm{III}})$. This implies that $\mathcal{W}_0^f(a^f)$ is either smaller than the lowest root of Equation (9) or greater than its highest root. Under Assumption 1, at $\mathcal{W}_2^f(a^f)$, $u(c_{\mathrm{m}}, n_{\mathrm{III}}) \geq u(c_{\mathrm{IV}}^f, 0)$ when $a < \overline{a}$, meaning that $\mathcal{W}_2^f(a^f)$ is in between the roots of Equation (9). As $\mathcal{W}_0^f(a^f) > \mathcal{W}_2^f(a^f)$, $\mathcal{W}_0^f(a^f)$ is above the highest root of Equation (9).

This means if $a^f < \underline{a}$, women are either involuntarily childless or voluntarily childless: once women become able to procreate ($w^f > \mathcal{W}_0^f(a^f)$), becoming a mother is not optimal as the opportunity cost of children is too high.

Proof and Implications of 2.

- First, we show that, for $a^f \in [\underline{a}, \overline{a}[$, $\mathcal{W}_0^f(a^f) < \mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f)$.

  Considering Equation (10), when $w^f = \mathcal{W}_2^f(a^f)$, we have

$$LHS(\mathcal{W}_2^f(a^f)) = \frac{2c_{\mathrm{m}} + (a^f - \mu)\phi(\nu - \eta)}{1 + \phi(\nu - \eta)}\nu$$

$$RHS(\mathcal{W}_2^f(a^f)) = \frac{c_{\mathrm{m}}}{2c_{\mathrm{m}} - (a^f - \mu)}\frac{1 + \phi(\nu - \eta)}{\phi}.$$

When $a^f \in [\underline{a}, \overline{a}[$, $RHS(\mathcal{W}_2^f(a^f)) > LHS(\mathcal{W}_2^f(a^f))$ is satisfied under Assumption 1. This ensures that $\mathcal{W}_2^f(a^f)$ is in between the two roots that solve Equation (9). As $\mathcal{W}_1^f(a^f)$, defined in Definition 2, is the smallest root of that equation, $\mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f)$.

$\mathcal{W}_0^f(a^f) < \mathcal{W}_2^f(a^f)$ when $a^f > \underline{a}$ and the Inequality (10) is not satisfied for $\mathcal{W}_0^f(a^f)$, then $\mathcal{W}_0^f(a^f) < \mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f)$.

Then, we can conclude that, under Assumption 1, $\forall w^f \in [\mathcal{W}_0^f(a^f), \mathcal{W}_1^f(a^f)[$, $u(c_{\mathrm{IV}}^f, 0) > u(c_{\mathrm{m}}, n_{\mathrm{III}})$ and that $\forall w^f \in [\mathcal{W}_1^f(a^f), \mathcal{W}_2^f(a^f)[$, $u(c_{\mathrm{m}}, n_{\mathrm{III}}) \geq u(c_{\mathrm{IV}}^f, 0)$.

- Second, we show that $\mathcal{W}_2^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$ when $a \in [\underline{a}, \overline{a}]$.

The value of $\mathcal{W}_2^f(a^f)$, as defined in Definition 2, solves $n_{\mathrm{III}} = n_{\mathrm{I}}$. At $\mathcal{W}_2^f(a^f)$, $u(c_{\mathrm{m}}, n_{\mathrm{III}}) = u(c_{\mathrm{I}}^f, n_{\mathrm{I}})$ and Regime I can be reached. This means that, $\forall w^f > \mathcal{W}_2^f(a^f)$, $u(c_{\mathrm{I}}^f, n_{\mathrm{I}}) > u(c_{\mathrm{m}}, n_{\mathrm{III}})$. At $\mathcal{W}_2^f(a^f)$, $u(c_{\mathrm{m}}, n_{\mathrm{III}}) \geq u(c_{\mathrm{IV}}^f, 0)$. So, we can conclude that $u(c_{\mathrm{I}}^f, n_{\mathrm{I}}) \geq u(c_{\mathrm{IV}}^f, 0)$ at $\mathcal{W}_2^f(a^f)$.

Regime I exists for $w^f \in [\mathcal{W}_2^f(a^f), \mathcal{W}_5^f(a^f)[$ where the value of $\mathcal{W}_5^f(a^f)$ as defined in Definition 2 solves $n_{\mathrm{I}} = 0$. For all $w^f > \mathcal{W}_5^f(a^f)$, $u(c_{\mathrm{I}}^f, n_{\mathrm{I}})$ is not defined since $\partial n_{\mathrm{I}}/\partial w^f < 0$, so that n_{I} would be negative. Then, for $w^f > \mathcal{W}_5^f(a^f)$, Regime IV prevails if single. Let us compare utility in Regime IV with utility in Regime I when $w^f \leq \mathcal{W}_5^f(a^f)$. $u(c_{\mathrm{IV}}^f, 0) \geq u(c_{\mathrm{I}}^f, n_{\mathrm{I}})$ if and only if,

$$\ln \frac{w^f + a^f - \mu}{w^f(1 + \phi(\nu - \eta)) + a^f - \mu} + \ln \nu \geq \ln \frac{w^f(1 + \phi(\nu - \eta)) + a^f - \mu}{\phi w^f} - 2 \ln 2. \quad (11)$$

Considering Equation (11), at $w^f = 0$, the LHS is equal to $\ln \nu$ and the RHS goes to $+\infty$. At $+\infty$, the limits of the LHS and the RHS are, respectively,

$$\ln \frac{1}{1 + \phi(\nu - \eta)} + \ln \nu$$

and

$$\ln \frac{1 + \phi(\nu - \eta)}{\phi} - 2 \ln 2$$

so that the RHS is above the LHS for low values of w^f. For large values of w^f, we cannot rank the two limits, and the RHS can be above or below the LHS. As both sides of the inequality are strictly decreasing and convex with w^f, the LHS can be equal to the RHS for either one or two values of w^f.

At $w^f = \mathcal{W}_5^f(a^f)$, the LHS is larger than the RHS (LHS - RHS $= \ln(1 + \eta/\nu)$), implying that $u(c_{\mathrm{IV}}^f, 0) > u(c_{\mathrm{I}}^f, n_{\mathrm{I}})$. This implies that $\mathcal{W}_5^f(a^f)$ is either in between the roots of Equation (11) holding at equality or at the right of the only root. Since $u(c^f_{\text{I}}, n_{\text{I}})$ is not defined for $w^f \geq \mathcal{W}^f_5(a^f)$, the relevant root $\mathcal{W}^f_4(a^f)$ of LHS=RHS is therefore for a value of w^f lower than $\mathcal{W}^f_5(a^f)$ and we have $\mathcal{W}^f_4(a^f) < \mathcal{W}^f_5(a^f)$. This proves that $u(c^f_{\text{IV}}, 0) < u(c^f_{\text{I}}, n_{\text{I}})$ for $w^f < \mathcal{W}^f_4(a^f)$, and that $u(c^f_{\text{IV}}, 0) \geq u(c^f_{\text{I}}, n_{\text{I}})$ for $w^f \in [\mathcal{W}^f_4(a^f), \mathcal{W}^f_5](a^f)$. Intuitively, because there is a fixed cost to becoming a parent, the optimal fertility is not continuous in w^f. At the point $\mathcal{W}^f_4(a^f)$, utility with a positive fertility is equal to utility with zero fertility.

We showed that $\forall w^f < \mathcal{W}^f_4(a^f)$, $u(c^f_{\text{I}}, n_{\text{I}}) > u(c^f_{\text{IV}}, 0)$ and that at $\mathcal{W}^f_2(a^f)$, Regime I exists and $u(c^f_{\text{I}}, n_{\text{I}}) > u(c^f_{\text{IV}}, 0)$. This implies that $\mathcal{W}^f_2(a^f) < \mathcal{W}^f_4(a^f)$.

Until now, we have proved that $\mathcal{W}^f_0(a^f) < \mathcal{W}^f_1(a^f) < \mathcal{W}^f_2(a^f) < \mathcal{W}^f_4(a^f) < \mathcal{W}^f_5(a^f)$ under Assumption 1 .

Regime V is not reachable when $a^f < \overline{a}$ since the consumption in Regime V is equal to $a^f - \mu$, which does not allow the woman to reach the minimal consumption level allowing her to procreate.

Summarizing these partial results, we can conclude that:

- $\forall w^f < \mathcal{W}^f_0(a^f)$, $c^f = c^f_{\text{II}}$, $n = 0$,
- $\forall w^f \in [\mathcal{W}^f_0(a^f), \mathcal{W}^f_1(a^f)[$, $u(c^f_{\text{IV}}, 0) > u(c_{\text{m}}, n_{\text{III}})$ and $c^f = c^f_{\text{IV}}$, $n = 0$,
- $\forall w^f \in [\mathcal{W}^f_1(a^f), \mathcal{W}^f_2(a^f)[$, $u(c_{\text{m}}, n_{\text{III}}) \geq u(c^f_{\text{IV}}, 0)$ and $c^f = c_{\text{m}}$, $n = n_{\text{III}}$,
- $\forall w^f \in [\mathcal{W}^f_2(a^f), \mathcal{W}^f_4(a^f)[$, $u(c^f_{\text{I}}, n_{\text{I}}) > u(c^f_{\text{IV}}, 0)$ and $c^f = c^f_{\text{I}}$, $n = n_{\text{I}}$,
- $\forall w^f \in [\mathcal{W}^f_4(a^f), \mathcal{W}^f_5(a^f)]$, $u(c^f_{\text{IV}}, 0) \geq u(c^f_{\text{I}}, n_{\text{I}})$ and $c^f = c^f_{\text{IV}}$, $n = n_{\text{IV}}$
- $\forall w^f \geq \mathcal{W}^f_5(a^f)$, $c^f = c^f_{\text{V}}$, $n = 0$.

Proof and Implications of 3.

We are now in the case $a^f \geq \overline{a}$, where Regime V is accessible for all $w^f \geq 0$. Furthermore, Regimes II and III no longer exist since even with a wage equal to zero, a woman can consume more than c_{m} allowing her to procreate. We then just need to compare the utilities in Regimes I, IV and V.

From Equation (7), $n_{\text{I}} > \bar{n}_{\text{M}} \iff w^f < \mathcal{W}^f_3(a^f)$, with $\mathcal{W}^f_3(a^f)$ defined in Definition 2 solving $n_{\text{I}} = \bar{n}_{\text{M}}$. For all $w^f < \mathcal{W}^f_3(a^f)$, $u(c^f_{\text{I}}, n_{\text{I}})$ is not defined as n_{I} would be above the maximum possible (more time in life would be needed).

Let us now show that $\mathcal{W}^f_3(a^f) < \mathcal{W}^f_4(a^f) < \mathcal{W}^f_5(a^f)$. We know from Inequation (11) that both the LHS and the RHS are strictly decreasing and convex in w^f. We can check that at $\mathcal{W}_3^f(a^f)$ the LHS is lower than the RHS. It follows from the definitions of $\mathcal{W}_3^f(a^f)$ and $\mathcal{W}_5^f(a^f)$ (Definition 2) that $\mathcal{W}_3^f(a^f) < \mathcal{W}_5^f(a^f)$. Since at $\mathcal{W}_4^f(a^f)$ the LHS is equal to the RHS, then $\mathcal{W}_3^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$.

At $\mathcal{W}_3^f(a^f)$ we can show that $u(c_{\mathrm{V}}^f, \bar{n}_{\mathrm{M}}) = u(c_{\mathrm{I}}^f, n_{\mathrm{I}}) > u(c_{\mathrm{IV}}^f, 0)$. As $u(c_{\mathrm{I}}^f, n_{\mathrm{I}})$ is increasing in w^f and $u(c_{\mathrm{V}}^f, \bar{n}_{\mathrm{M}})$ is unaffected by w^f, we have that $u(c_{\mathrm{I}}^f, n_{\mathrm{I}}) \geq u(c_{\mathrm{V}}^f, \bar{n}_{\mathrm{M}})$ for $w^f \in [\mathcal{W}_3^f(a^f), \mathcal{W}_5^f(a^f)]$. As $u(c_{\mathrm{IV}}^f, 0)$ is increasing in w^f and $u(c_{\mathrm{V}}^f, \bar{n}_{\mathrm{M}})$ is unaffected by w^f, we also have that $u(c_{\mathrm{V}}^f, \bar{n}_{\mathrm{M}}) > u(c_{\mathrm{IV}}^f, 0)\ \forall w^f < \mathcal{W}_3^f(a^f)$.

Given that $\mathcal{W}_3^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$, we can conclude that:

- when $w^f \leq \mathcal{W}_4^f(a^f)$, $c^f = c_{\mathrm{V}}$ and $n = \bar{n}_{\mathrm{M}}$,
- when $w^f \in [\mathcal{W}_4^f(a^f), \mathcal{W}_5^f(a^f)]$, $u(c_{\mathrm{IV}}^f, 0) \geq u(c_{\mathrm{I}}^f, n_{\mathrm{I}}) > u(c_{\mathrm{V}}^f, \bar{n}_{\mathrm{M}})$, which implies that $c^f = c_{\mathrm{I}}^f$ and $n = n_{\mathrm{I}}$,
- when $w^f > \mathcal{W}_5^f(a^f)$, $u(c_{\mathrm{IV}}^f, 0) > u(c_{\mathrm{V}}^f, \bar{n}_{\mathrm{M}})$ and $c^f = c_{\mathrm{V}}^f$ while $n = 0$.

■

B.2 Optimal Decisions of Couples

As for single women, we provide some wage thresholds as well as non-labor income thresholds that are useful to describe the optimal decisions of couples which will be described in the following appendix. We also determine Assumption 2 under which our results will be valid. We then state Proposition 3, where we describe the optimal decisions of a couple as a function of its labor and non-labor incomes, and prove it.

Definition 3 (Wage thresholds for couples)

$$\mathcal{W}_A^f(a) = \frac{c_m - a}{1 - \alpha\phi\eta}, \quad \mathcal{W}_B^f(a) = \frac{c_m - a}{1 - \alpha\phi(\eta + \frac{\nu\underline{\theta}}{2})},$$

$$\mathcal{W}_D^f(a) = \frac{2c_m - a\left(1 - \frac{1}{2}\underline{\theta}\right)}{\left(1 - \frac{1}{2}\underline{\theta}\right)\left(1 + \alpha\phi(\nu - \eta)\right)}, \quad \mathcal{W}_E^f(a) = \frac{a - c_m}{\frac{1}{2}\underline{\theta}\left(1 + \alpha\phi(\nu - \eta)\right)},$$

$$\mathcal{W}_F^f(a) = \frac{a}{1 + \alpha\phi(\nu - \eta)}, \quad \mathcal{W}_I^f(a) \equiv \frac{1 - \phi\alpha\eta}{\phi\alpha\nu}a,$$

$\mathcal{W}_C^f(a)$ is such that $U(c_{VIII}^m, c_{VIII}^f, n_{VII}) - U(c_{IX}^m, c_{IX}^f, 0)$ is maximal.
$\mathcal{W}_G^f(a)$ is such that $U(c_X^m, c_m, n_M) = U(c_{IX}^m, c_{IX}^f, 0)$.

$\mathcal{W}^f_H(a)$ *is the highest root of the equation* $U(c^m_{VI}, c^f_{VI}, n_{VI}) = U(c^m_{IX}, c^f_{IX}, 0)$.
$\mathcal{W}^f_{\underline{C}}(a)$ *and* $\mathcal{W}^f_{\overline{C}}(a)$ *satisfy* $U(c^m_{VIII}, c^f_{VIII}, n_{VIII}) = U(c^m_{IX}, c^f_{IX}, 0)$.

Note: $\mathcal{W}^f_A(a)$ is such that the couple can afford positive fertility ($n \to 0$, $c^f = c_{\mathrm{m}}$ and $c^m = 0$). $\mathcal{W}^f_B(a)$ is such that $n_{\mathrm{VIII}} = 0$. $\mathcal{W}^f_D(a)$ is such that $n_{\mathrm{VIII}} = n_{\mathrm{VI}}$. $\mathcal{W}^f_E(a)$ is such that $n_{\mathrm{VIII}} = n_{\mathrm{M}}$. $\mathcal{W}^f_F(a)$ is such that $n_{\mathrm{VI}} = n_{\mathrm{M}}$. $\mathcal{W}^f_I(a)$ is such that $U(c^m_{\mathrm{XI}}, c^f_{\mathrm{XI}}, n_{\mathrm{M}}) = U(c^m_{\mathrm{IX}}, c^f_{\mathrm{IX}}, 0)$.

$\mathcal{W}^f_C(a)$ is the value of w^f that maximizes $U(c^m_{\mathrm{VII}}, c^f_{\mathrm{VII}}, n_{\mathrm{VII}}) - U(c^m_{\mathrm{IX}}, c^f_{\mathrm{IX}}, 0)$:

$$\mathcal{W}^f_C(a) = \frac{-[a(1+\alpha\phi(\nu-\eta))\frac{1}{2}\underline{\theta} + 2(c_{\mathrm{m}} - a)]}{2\left(\frac{1}{2}\underline{\theta} - 1\right)(1+\alpha\phi(\nu-\eta))} - \frac{\sqrt{[a(1+\alpha\phi(\nu-\eta))\frac{1}{2}\underline{\theta} - 2(a-c_{\mathrm{m}})]^2 + 4a(a-c_{\mathrm{m}})\left(\frac{1}{2}\underline{\theta} - 1\right)(1+\alpha\phi(\nu-\eta))}}{2\left(\frac{1}{2}\underline{\theta} - 1\right)(1+\alpha\phi(\nu-\eta))}.$$

$\mathcal{W}^f_{\underline{C}}(a)$ and $\mathcal{W}^f_{\overline{C}}(a)$ are respectively the lowest and the highest roots of

$$\ln\left(\alpha\phi\nu w^f(w^f+a)\right) = \left(1 - \frac{1}{2}\underline{\theta}\right)\ln\frac{c_{\mathrm{m}}}{1-\frac{1}{2}\underline{\theta}} + \left(1 + \frac{1}{2}\underline{\theta}\right)\ln\frac{w^f(1+\alpha\phi(\nu-\eta)) + a - c_{\mathrm{m}}}{1+\frac{1}{2}\underline{\theta}}.$$

$\mathcal{W}^f_G(a)$ is such that $U(c^m_{\mathrm{X}}, c_{\mathrm{m}}, n_{\mathrm{M}}) = U(c^m_{\mathrm{IX}}, c^f_{\mathrm{IX}}, 0)$:

$$\mathcal{W}^f_G(a) = \left(\frac{c_{\mathrm{m}}}{1-\frac{1}{2}\underline{\theta}}\right)^{1-\frac{1}{2}\underline{\theta}} \left(\frac{a-c_{\mathrm{m}}}{\frac{1}{2}\underline{\theta}}\right)^{\frac{1}{2}\underline{\theta}} \frac{1+\alpha\phi(\nu-\eta)}{\alpha\phi\nu} - a.$$

$\mathcal{W}^f_H(a)$ is the highest root of the equation $U(c^m_{\mathrm{VI}}, c^f_{\mathrm{VI}}, n_{\mathrm{VI}}) = U(c^m_{\mathrm{IX}}, c^f_{\mathrm{IX}}, 0)$:

$$\mathcal{W}^f_H(a) = \frac{\alpha\phi(\nu+\eta) - 1 + \sqrt{(\alpha\phi(\nu+\eta)-1)^2 + (4\alpha\phi\nu - (1+\alpha\phi(\nu-\eta)))^2)}}{a},$$

Definition 4 (Non-labor income thresholds for couples)
A_0 *is the value of* a *that solves* $U(c^m_{VII}, c^f_{VII}, n_{VII}) = U(c^m_{IX}, c^f_{IX}, 0)$ *where* $w^f = \mathcal{W}^f_C(a)$.
A_1 *is the value of* a *that solves* $U(c^m_{VII}, c^f_{VII}, n_{VII}) = U(c^m_{IX}, c^f_{IX}, 0)$ *where* $w^f = \mathcal{W}^f_D(a)$.
$A_2 = c_m$,
A_3 *is the lowest value of* a *such that* $\mathcal{W}^f_G(a) = 0$,
A_4 *is the value of* a *such that* $\mathcal{W}^f_G(a) = \mathcal{W}^f_I(a)$,
$A_5 = \dfrac{c_m}{1-\frac{1}{2}\underline{\theta}}$.

The following assumption is a technical assumption on the parameters (see Appendix B.2 for details).

Assumption 2 *Parameters α, ν, η, $\underline{\theta}$, and c_m satisfy that:*

when $w^f = \mathcal{W}^f_C(a)$, $U(c_m, c^m_{VIII}, n_{VIII}) - U(c^f_{IX}, c^m_{IX}, 0)$ increases with $a, \forall a < A_4$;

$\mathcal{W}^f_G(A_5) > \mathcal{W}^f_E(A_5)$ and $(\partial \mathcal{W}^f_G(a) - \mathcal{W}^f_E(a))/\partial a > 0$ $\forall a \leq A_5$.

Details for Assumption 2:

$$\forall a < A_4 : \phi\alpha\nu \left[\frac{\partial \mathcal{W}^f_C(a)}{\partial a}(2\mathcal{W}^f_C(a) + a) + \mathcal{W}^f_C(a) \right] <$$

$$\left(\frac{c_{\mathrm{m}}}{1 - \frac{1}{2}\underline{\theta}} \right)^{1-\frac{1}{2}\underline{\theta}} \frac{1 + \phi\alpha(\nu - \eta)}{(1 + \frac{1}{2}\underline{\theta})^{\frac{1}{2}\underline{\theta}}} \left((1 + \phi\alpha(\nu - \eta))\mathcal{W}^f_C(a) + a - c_{\mathrm{m}} \right)^{\frac{1}{2}\underline{\theta}} \frac{\partial \mathcal{W}^f_C(a)}{\partial a},$$

implies that $w^f = \mathcal{W}^f_C(a)$, $U(c_{\mathrm{m}}, c^m_{\mathrm{VIII}}, n_{\mathrm{VIII}}) - U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, 0)$ is increasing in $a, \forall a < A_4$.

$$(1 - \frac{1}{2}\underline{\theta})^{1-\frac{1}{2}\underline{\theta}} \frac{(1 + \phi\alpha(\nu - \eta))^2}{\phi\alpha\nu} > \max \left\{ \frac{1 + \phi\alpha(\nu - \eta)}{1 - \frac{1}{2}\underline{\theta}}; \left[1 + (1 - \frac{1}{2}\underline{\theta})(1 + \phi\alpha(\nu - \eta)) \right] c_{\mathrm{m}}^{\frac{1}{2}\underline{\theta}} \right\}.$$

implies that $\mathcal{W}^f_G(A_5) > \mathcal{W}^f_E(A_5)$ and $(\partial \mathcal{W}^f_G(a) - \mathcal{W}^f_E(a))/\partial a > 0$ $\forall a \leq A_5$.

Proposition 3 *Under Assumption 2, the optimal choice of a couple with non-labor income a and wages w^f is given by:*

1. *When $a \leq A_0$:*
   - *if $w^f < \mathcal{W}^f_A(a)$, $c^f = c^f_{VII}$, $c^m = c^m_{VII}$, $n = 0$*
   - *if $w^f \geq \mathcal{W}^f_A(a)$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$*
2. *When $a \in]A_0, A_1]$:*
   - *if $w^f < \mathcal{W}^f_A(a)$, $c^f = c^f_{VII}$, $c^m = c^m_{VII}$, $n = 0$*
   - *if $w^f \in]\mathcal{W}^f_A(a), \mathcal{W}^f_{\underline{C}}(a)]$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$*
   - *if $w^f \in]\mathcal{W}^f_{\underline{C}}(a), \mathcal{W}^f_{\overline{C}}(a)]$, $c^f = c^f_{VIII}$, $c^m = c^m_{VIII}$, $n = n_{VIII}$*
   - *if $w^f \geq \mathcal{W}^f_{\overline{C}}(a)$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$*
3. *When $a \in]A_1, A_2]$:*
   - *if $w^f < \mathcal{W}^f_A(a)$, $c^f = c^f_{VII}$, $c^m = c^m_{VII}$, $n = 0$*
   - *if $w^f \in]\mathcal{W}^f_A(a), \mathcal{W}^f_{\underline{C}}(a)]$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$*

- *if* $w^f \in]\mathcal{W}^f_{\underline{C}}(a), \mathcal{W}^f_D(a)]$, $c^f = c^f_{VIII}$, $c^m = c^m_{VIII}$, $n = n_{VIII}$
- *if* $w^f \in]\mathcal{W}^f_D(a), \mathcal{W}^f_H(a)[$, $c^f = c^f_{VI}$, $c^m = c^m_{VI}$, $n = n_{VI}$
- *if* $w^f \geq \mathcal{W}^f_H(a)$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

4. *When* $a \in]A_2, A_3]$*:*

- *if* $w^f \leq \mathcal{W}^f_{\underline{C}}(a)$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$
- *if* $w^f \in]\mathcal{W}^f_{\underline{C}}(a), \mathcal{W}^f_D(a)[$, $c^f = c^f_{VIII}$, $c^m = c^m_{VIII}$, $n = n_{VIII}$
- *if* $w^f \in]\mathcal{W}^f_D(a), \mathcal{W}^f_H(a)[$, $c^f = c^f_{VI}$, $c^m = c^m_{VI}$, $n = n_{VI}$
- *if* $w^f \geq \mathcal{W}^f_H(a)$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

5. *When* $a \in]A_3, A_4]$*:*

- *if* $w^f \leq \mathcal{W}^f_G(a)$, $c^f = c^f_X$, $c^m = c^m_X$, $n = n_M$
- *if* $w^f \in]\mathcal{W}^f_G(a), \max\{\mathcal{W}^f_E(a), \mathcal{W}^f_{\underline{C}}(a)\}[$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$
- *if* $w^f \in]\max\{\mathcal{W}^f_E(a), \mathcal{W}^f_{\underline{C}}(a)\}, \mathcal{W}^f_D(a)[$, $c^f = c^f_{VIII}$, $c^m = c^m_{VIII}$, $n = n_{VIII}$
- *if* $w^f \in]\mathcal{W}^f_D(a), \mathcal{W}^f_H(a)[$, $c^f = c^f_{VI}$, $c^m = c^m_{VI}$, $n = n_{VI}$
- *if* $w^f \geq \mathcal{W}^f_H(a)$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

6. *When* $a \in]A_4, A_5]$*:*

- *if* $w^f \leq \mathcal{W}^f_E(a)$, $c^f = c^f_X$, $c^m = c^m_X$, $n = n_M$
- *if* $w^f \in]\mathcal{W}^f_E(a), \mathcal{W}^f_D(a)[$, $c^f = c^f_{VIII}$, $c^m = c^m_{VIII}$, $n = n_{VIII}$
- *if* $w^f \in]\mathcal{W}^f_D(a), \mathcal{W}^f_H(a)[$, $c^f = c^f_{VI}$, $c^m = c^m_{VI}$, $n = n_{VI}$
- *if* $w^f \geq \mathcal{W}^f_H(a)$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

7. *When* $a > A_5$*:*

- *if* $w^f \leq \mathcal{W}^f_F(a)$, $c^f = c^f_{XI}$, $c^m = c^m_{XI}$, $n = n_M$
- *if* $w^f \in]\mathcal{W}^f_F(a), \mathcal{W}^f_H(a)[$, $c^f = c^f_{VI}$, $c^m = c^m_{VI}$, $n = n_{VI}$
- *if* $w^f \geq \mathcal{W}^f_H(a)$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

Proof. We use a two-step strategy. First, we highlight some general properties of the model. Second, we prove each part of the proposition.

Step 1: Some General Properties of the Model

- *Regimes VII and IX are "continuous"*

  $\forall w^f > 0$, $U(c^f_{\rm VII}, c^m_{\rm VII}, 0) = U(c^f_{\rm IX}, c^m_{\rm IX}, 0)$. A couple is able to have children once its total income is greater than $c_{\rm m}$ which is satisfied when $w^f > \mathcal{W}^f_A(a)$. This implies that $\forall w^f < \mathcal{W}^f_A(a)$, childlessness is involuntary, while it is voluntary above this threshold.

- *Regimes VIII and VI are "continuous"*

  Regime VIII exists once $n_{\rm VIII} \geq 0$ which is satisfied $\forall w^f \geq \mathcal{W}^f_B(a)$, while Regime VI exists once $c^f_{\rm VI} \geq c_{\rm m}$ which is satisfied $\forall w^f \geq \mathcal{W}^f_D(a)$. We can verify that when $w^f = \mathcal{W}^f_D(a)$, $c^f_{\rm VI} = c_{\rm m}$ and $n_{\rm VI} = n_{\rm VIII}$, it induces $U(c_{\rm m}, c^m_{\rm VIII}, n_{\rm VIII}) = U(c^f_{\rm VI}, c^m_{\rm VI}, n_{\rm VI})$ for $w^f = \mathcal{W}^f_D(a)$. It also implies that Regime VIII is defined for all $w^f \in [\mathcal{W}^f_B(a), \mathcal{W}^f_D(a)]$ while Regime VI is defined $\forall w^f \geq \mathcal{W}^f_D(a)$ ($\lim_{w^f \to \infty} n_{\rm VI} > 0$).

- *Equation $U(c^f_{VI}, c^m_{VI}, n_{VI}) = U(c^f_{IX}, c^m_{IX}, 0)$ always admits two positive roots when it is solved with respect to w^f in $\mathbb{R}$.*

  $U(c^f_{\rm VI}, c^m_{\rm VI}, n_{\rm VI}) = U(c^f_{\rm IX}, c^m_{\rm IX}, 0)$

  $$\Leftrightarrow \left[4\nu\alpha\phi w^f(w^f + a)\right]^{\frac{1}{2}} = w^f(1 + \alpha\phi(\nu - \eta)) + a. \tag{12}$$

  Let us denote by $LHS(w^f)$ the left-hand side of Equation (12) and by $RHS(w^f)$ its right-hand side. We have that $LHS(0) = 0$ while $RHS(0) = a > 0$. Furthermore, $RHS(w^f)$ is increasing and linear while $LHS(w^f)$ is increasing and concave:

  $$\begin{aligned} \frac{\partial LHS(w^f)}{\partial w^f} &= 2\phi\alpha\nu(2w^f + a)(w^f[w^f + a])^{-\frac{1}{2}} > 0 \\ \frac{\partial^2 LHS(w^f)}{\partial w^{f^2}} &= -\phi\alpha\nu[w^f(w^f + a)]^{-\frac{3}{2}} a^2 < 0. \end{aligned}$$

  Because of the respective curvatures of LHS and RHS and because $LHS(0) < RHS(0)$, Equation (12) admits at most two positive roots. These solutions can be found by solving the quadratic equation:

  $$[4\phi\alpha\nu - (1 + \phi\alpha(\nu - \eta))^2]w^{f^2} + 2aw^f[\phi\alpha(\nu + \eta) - 1] - a^2 = 0.$$

  As the discriminant of this equation equals $R \equiv 16\nu\eta(\phi\alpha a)^2 > 0$, Equation (12) admits two real roots that we denote respectively $\mathcal{W}^f_{\underline{H}}$ and $\mathcal{W}^f_{\overline{H}}$:

  $$\mathcal{W}^f_{\underline{H}} = \frac{1 - \phi\alpha(\nu + \eta) + 2a\alpha\phi\sqrt{\eta\nu}}{4\phi\alpha\nu - (1 + \phi\alpha(\nu - \eta))^2} \quad , \quad \mathcal{W}^f_{\overline{H}} = \frac{1 - \phi\alpha(\nu + \eta) - 2a\alpha\phi\sqrt{\eta\nu}}{4\phi\alpha\nu - (1 + \phi\alpha(\nu - \eta))^2}.$$

Necessarily, $(\mathcal{W}^f_{\underline{H}}, \mathcal{W}^f_{\overline{H}}) > (0,0)$. As $\phi\alpha(\nu+\eta)-1>0$, $(4\phi\alpha\nu-(1+\phi\alpha(\nu-\eta))^2)$ has to be negative to ensure that $\mathcal{W}^f_{\overline{H}} > 0$. We can then deduce that $\mathcal{W}^f_{\overline{H}} > \mathcal{W}^f_{\underline{H}} > 0$. Both $\mathcal{W}^f_{\overline{H}}$ and $\mathcal{W}^f_{\underline{H}}$ are linearly increasing with a and $\mathcal{W}^f_{\underline{H}} = \mathcal{W}^f_{\overline{H}} = 0$ when $a=0$.

- *When $a^f \leq A_0$, $U(c^f_{VII}, c^m_{VII}, 0)$ and $U(c^f_{IX}, c^m_{IX}, 0)$ are always higher than $U(c_m, c^m_{VIII}, n_{VIII})$, and also than $U(c^f_{VI}, c^m_{VI}, n_{VI})$ for $w^f = \mathcal{W}^f_D(a)$.*

  $U(c_{\mathrm{m}}, c^m_{\mathrm{VIII}}, n_{\mathrm{VIII}})$ and $U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, 0)$ are both increasing in a.
  $U(c_{\mathrm{m}}, c^m_{\mathrm{VIII}}, n_{\mathrm{VIII}}) - U(c^m_{\mathrm{IX}}, c^f_{\mathrm{IX}}, 0)$ is positive when,

$$\left(\alpha\phi\nu w^f(w^f+a)\right)^{\frac{1}{1+\frac{1}{2}\underline{\theta}}} > \left(\frac{c_{\mathrm{m}}}{1-\frac{1}{2}\underline{\theta}}\right)^{\frac{1-\frac{1}{2}\underline{\theta}}{1+\frac{1}{2}\underline{\theta}}} \frac{w^f(1+\alpha\phi(\nu-\eta))+a-c_{\mathrm{m}}}{1+\frac{1}{2}\underline{\theta}}. \tag{13}$$

  Since the left-hand-side is strictly increasing and concave with respect to w^f and the right-hand-side is linearly increasing with w^f, we know that $U(c_{\mathrm{m}}, c^m_{\mathrm{VIII}}, n_{\mathrm{VIII}}) - U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, 0)$ admits a unique maximum that we denote $\mathcal{W}_C(a)$. Furthermore, we can check that Inequality (13) is not satisfied when $w^f=0$ and $a<c_{\mathrm{m}}$.

  Under Assumption 2, when $w^f = \mathcal{W}^f_C(a)$, $U(c_{\mathrm{m}}, c^m_{\mathrm{VIII}}, n_{\mathrm{VIII}}) - U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, 0)$ is increasing in a for $a<A_4$. This means that the maximum value of $U(c_{\mathrm{m}}, c^m_{\mathrm{VIII}}, n_{\mathrm{VIII}}) - U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, 0)$ increases monotonically with a. It implies that there is a value, $a=A_0$, where A_0 solves $U(c_{\mathrm{m}}, c^m_{\mathrm{VIII}}, n_{\mathrm{VIII}}) = U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, 0)$ when $w^f = \mathcal{W}^f_C(a)$:

$$\begin{aligned}&\left(1+\frac{1}{2}\underline{\theta}\right)\ln\frac{c_{\mathrm{m}}}{1+\frac{1}{2}\underline{\theta}} - \left(1-\frac{1}{2}\underline{\theta}\right)\ln\left(1-\frac{1}{2}\underline{\theta}\right)\\ &+\left(1+\frac{1}{2}\underline{\theta}\right)\ln\left(\mathcal{W}^f_C(1+\alpha\phi(\nu-\eta))+A_0-c_{\mathrm{m}}\right) - \ln\left(\alpha\phi\nu\mathcal{W}^f_C\left(\mathcal{W}^f_C+A_0\right)\right)=0.\end{aligned}$$

  This means that when $a^f \leq A_0$, $U(c^f_{\mathrm{VII}}, c^m_{\mathrm{VII}}, 0) = U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, 0) > U(c_{\mathrm{m}}, c^m_{\mathrm{VIII}}, n_{\mathrm{VIII}})$. As $w^f = \mathcal{W}^f_D(a)$ implies that $U(c^f_{\mathrm{VI}}, c^m_{\mathrm{VI}}, n_{\mathrm{VI}}) = U(c^f_{\mathrm{VIII}}, c^m_{\mathrm{VIII}}, n_{\mathrm{VIII}})$, we also find that $U(c^f_{\mathrm{VI}}, c^m_{\mathrm{VI}}, n_{\mathrm{VI}}) < U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, n_{\mathrm{IX}})$ for $w^f = \mathcal{W}^f_D(a)$ and $a \leq A_0$.

- *$U(c^f_{VI}, c^m_{VI}, n_{VI}) < U(c^f_{IX}, c^m_{IX}, 0)$ for all $w^f > \mathcal{W}^f_D(a)$ when $a \leq A_1$*

  The equation $U(c^f_{\mathrm{VI}}, c^m_{\mathrm{VI}}, n_{\mathrm{VI}}) - U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, 0)$ is quadratic with respect to w^f. It admits a unique maximum for $w^f = \mathcal{W}^f_J(a)$,

$$\mathcal{W}^f_J(a) \equiv \frac{\alpha\phi(\nu+\eta)-1}{4\alpha\phi\nu-(1+\alpha\phi(\nu-\eta))^2}a < \mathcal{W}^f_D(a).$$

  Then $U(c^f_{\mathrm{VI}}, c^m_{\mathrm{VI}}, n_{\mathrm{VI}}) - U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, 0)$ is decreasing for all $w^f > \mathcal{W}^f_D(a)$. Since $\forall a \leq A_1$,

$U(c^f_{\text{IX}}, c^m_{\text{IX}}, 0) > U(c^f_{\text{VIII}}, c^m_{\text{VIII}}, n_{\text{VIII}}) = U(c^f_{\text{VI}}, c^m_{\text{VI}}, n_{\text{VI}})$ for $w^f = \mathcal{W}^f_D(a)$, then $U(c^f_{\text{VI}}, c^m_{\text{VI}}, n_{\text{VI}}) < U(c^f_{\text{IX}}, c^m_{\text{IX}}, 0)$ for all $w^f > \mathcal{W}^f_D(a)$ when $a \leq A_1$.

- *Regime X is never preferred to Regime IX for $a < A_3$*

  $\mathcal{W}^f_G(a)$, as defined in Definition 3, is the unique root of $U(c^f_{\text{X}}, c_{\text{m}}, n_{\text{M}}) - U(c^f_{\text{IX}}, c^m_{\text{IX}}, 0) = 0$. As $\mathcal{W}^f_G(a) < 0$ for $a < A_3$, Regime X will never be preferred to Regime IX for $a < A_3$.

- *There exists a unique $A_4 > 0$ such that $\forall a \in [A_3, A_4]$, $\mathcal{W}^f_G(a) < \mathcal{W}^f_E(a)$ while $\forall a \in [A_4, A_5]$, $\mathcal{W}^f_G(a) > \mathcal{W}^f_E(a)$*

  Here, we compute the values of $\mathcal{W}^f_G(a)$ and $\mathcal{W}^f_E(a)$ when a is equal either to $A_2 = c_{\text{m}}$ or A_5:

$$\mathcal{W}^f_G(c_{\text{m}}) \;=\; -c_{\text{m}} < 0 \quad , \quad \mathcal{W}^f_G(A_5) = \frac{1}{1-\frac{1}{2}\underline{\theta}} c_{\text{m}} \left[(1 - \frac{1}{2}\underline{\theta})^{\frac{1}{2}\underline{\theta}} \frac{1+\phi\alpha(\nu-\eta)}{\phi\alpha\nu} - 1 \right] > 0$$
$$\mathcal{W}^f_E(c_{\text{m}}) \;=\; 0 \quad , \quad \mathcal{W}^f_E(A_5) = \frac{c_{\text{m}}}{1+\phi\alpha(\nu-\eta)}.$$

  Under Assumption 2, $\mathcal{W}^f_G(A_5) > \mathcal{W}^f_E(A_5)$ and $\frac{\partial \mathcal{W}^f_G(a) - \mathcal{W}^f_E(a)}{\partial a} > 0 \; \forall a \leq A_5$. This implies that there exists a unique $A_4 > 0$ such that $\forall a \in [A_3, A_4]$, $\mathcal{W}^f_G(a) < \mathcal{W}^f_E(a)$ while $\forall a \in [A_4, A_5]$, $\mathcal{W}^f_G(a) > \mathcal{W}^f_E(a)$

Step 2: Proof of Each Part of the Proposition

Step 2.1: Cases where $a < A_2 = c_m$

Regimes X and XI are not reachable because $c^m_{\text{X}} < 0$ and $c^f_{\text{XI}} < c_{\text{m}}$ for $a^f < c_{\text{m}}$. So we only have to compare the utilities $U_{\text{VII}}(a, w^f)$, $U_{\text{VIII}}(a, w^f)$, $U_{\text{VI}}(a, w^f)$ and $U_{\text{IX}}(a, w^f)$ to see the outcome of a marriage.

Case 1: $a < A_0$

Here, we will show that, as in the first case of Proposition 1, once agents with low non-labor income are able to be parents, the opportunity cost of childrearing is too high for them to be willing to.

The value $\mathcal{W}^f_A$ defined in Definition 3 solves the following equation:

$$c_{\text{m}} + \alpha\phi\eta w^f = w^f + a.$$

$A_0 < c_{\rm m}$ so $\mathcal{W}_A^f > 0$. For $w^f < \mathcal{W}_A^f$, the consumption of a woman who wants to have children will be lower than $c_{\rm m}$ even if her husband does not consume. Consequently, for $w^f < \mathcal{W}_A^f$, the couple is involuntarily childless. For $w^f \geq \mathcal{W}_A^f$, the couple can have children, and has to decide between parenthood and childlessness.

As $a < A_0$, $U(c^f_{\rm VIII}, c^m_{\rm VIII}, n_{\rm VIII})$ is lower than $U(c^f_{\rm IX}, c^m_{\rm IX}, 0)$, $\forall w^f$. By continuity we know that $U(c^f_{\rm VI}, c^m_{\rm VI}, n_{\rm VI})$ is lower than $U(c^f_{\rm IX}, c^m_{\rm IX}, 0)$ for $w^f = \mathcal{W}^f_D(a)$. By definition of $\mathcal{W}^f_H(a)$, we know that $U(c^f_{\rm VI}, c^m_{\rm VI}, n_{\rm VI}) < U(c^f_{\rm IX}, c^m_{\rm IX}, 0)$ $\forall w^f > \mathcal{W}^f_H(a)$. As $\mathcal{W}^f_D(a) > \mathcal{W}^f_H(a)$, $\forall w^f \geq \mathcal{W}^f_A$, couples are voluntarily childless.

We can then conclude that for $w^f < \mathcal{W}^f_A(a)$, $c^f = c^f_{\rm VII}$, $c^m = c^m_{\rm VII}$, $n = 0$ while, if $w^f \geq \mathcal{W}^f_A(a)$, $c^f = c^f_{\rm IX}$, $c^m = c^m_{\rm IX}$, $n = 0$.

<u>**Case 2:** $a \in [A_0, A_1[$</u>

In this case, we will show that Malthusian mechanisms can be at play for some poor couples. When $a = A_1$ and $w^f = \mathcal{W}^f_D$, $U(c^f_{\rm VIII}, c^m_{\rm VIII}, n_{\rm VIII}) = U(c^f_{\rm VI}, c^m_{\rm VI}, n_{\rm VI}) = U(c^f_{\rm IX}, c^m_{\rm IX}, 0)$. We also know that for $w^f = \mathcal{W}^f_D$, $U(c^f_{\rm VI}, c^m_{\rm VI}, n_{\rm VI}) - U(c^f_{\rm IX}, c^m_{\rm IX}, 0)$ is negative when $a < A_1$.

As $A_1 < c_{\rm m}$, $\mathcal{W}^f_A > 0$ and couples are involuntarily childless when $w^f < \mathcal{W}^f_A$. Furthermore:

$$\lim_{w^f \to \mathcal{W}^f_A} U(c^f_{\rm VIII}, c^m_{\rm VIII}, n_{\rm VIII}) = -\infty.$$

This is because for $w^f \to \mathcal{W}^f_A$, $c^m_{\rm VIII} \to 0$. $U(c^f_{\rm VIII}, c^m_{\rm VIII}, n_{\rm VIII})$ is then lower than $U(c^f_{\rm IX}, c^m_{\rm IX}, 0)$ in the neighborhood of $\mathcal{W}^f_A$.

As $a > A_0$, $U(c^f_{\rm VIII}, c^m_{\rm VIII}, n_{\rm VIII}) - U(c^f_{\rm IX}, c^m_{\rm IX}, 0)$ admits two positive roots, denoted $\mathcal{W}^f_{\underline{C}}(a)$ and $\mathcal{W}^f_{\overline{C}}(a)$, and is positive between them. As $a \leq A_1$, when $w^f = \mathcal{W}^f_D(a)$, $U(c^f_{\rm VIII}, c^m_{\rm VIII}, n_{\rm VIII})$ $= U(c^f_{\rm VI}, c^m_{\rm VI}, n_{\rm VI}) < U(c^f_{\rm IX}, c^m_{\rm IX}, 0)$. This means that once the interior regime can be reached by the couple, having children is not optimal.

Since $\mathcal{W}^f_D(a) > \mathcal{W}^f_H(a)$, $U(c^f_{\rm VI}, c^m_{\rm VI}, n_{\rm VI}) - U(c^f_{\rm IX}, c^m_{\rm IX}, 0)$ is decreasing $\forall w^f > \mathcal{W}^f_D(a)$. This implies that $U(c^f_{\rm VI}, c^m_{\rm VI}, n_{\rm VI})$ is always smaller than $U(c^f_{\rm IX}, c^m_{\rm IX}, 0)$, $\forall w^f > \mathcal{W}^f_D(a)$. It also implies that although having children in the interior regime is possible for the couple, this will never be an optimal choice.

We conclude that for $a \in]A_0, A_1]$, couples are in Regime VII for $w^f < \mathcal{W}^f_A(a)$, in Regime IX for $w^f \in]\mathcal{W}^f_A(a), \mathcal{W}^f_{\underline{C}}(a)]$, in Regime VIII for $w^f \in]\mathcal{W}^f_{\underline{C}}(a), \mathcal{W}^f_{\overline{C}}(a)]$ and in Regime IX again for $w^f > \mathcal{W}^f_{\overline{C}}(a)$.

<u>**Case 3:** $a \in]A_1, A_2]$</u>

In this case, we can show that for intermediary values of non-labor incomes, a couple can either be in a Malthusian or a modern fertility pattern. This will crucially depend on the wage of the wife.

As $a \leq A_2 \equiv c_{\mathrm{m}}$, $\mathcal{W}_A^f(a) \geq 0$ and couples are involuntarily childless for $w^f < \mathcal{W}_A^f$. Using the same reasoning as for the previous case, $\lim_{w^f \to \mathcal{W}_A^f} U(c^f_{\mathrm{VIII}}, c^m_{\mathrm{VIII}}, n_{\mathrm{VIII}}) = -\infty$, and $U(c^f_{\mathrm{VIII}}, c^m_{\mathrm{VIII}}, n_{\mathrm{VIII}}) < U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, 0)$ in the neighborhood of $\mathcal{W}_A^f$.

Since $a > A_1$, $U(c^f_{\mathrm{VI}}, c^m_{\mathrm{VI}}, n_{\mathrm{VI}})$ is higher than $U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, 0)$ for $w^f = \mathcal{W}_D^f(a)$.
Since $U(c^f_{\mathrm{VIII}}, c^m_{\mathrm{VIII}}, n_{\mathrm{VIII}}) = U(c^f_{\mathrm{VI}}, c^m_{\mathrm{VI}}, n_{\mathrm{VI}})$ for $w^f = \mathcal{W}_D^f(a)$, we know that $\mathcal{W}_D^f(a)$ lies between $\mathcal{W}_{\underline{C}}^f(a)$ and $\mathcal{W}_{\overline{C}}^f(a)$. Furthermore, as $U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, n_{\mathrm{IX}}) - U(c^f_{\mathrm{VI}}, c^m_{\mathrm{VI}}, n_{\mathrm{VI}}) = 0$ admits two positive roots, $\mathcal{W}_{\underline{H}}^f$ and $\mathcal{W}_{\overline{H}}^f$, $\mathcal{W}_D^f(a)$ also lies between these two roots.

We conclude that for $a \in]A_1, A_2]$, couples are in Regime VII when $w^f < \mathcal{W}_A^f(a)$, in Regime IX when $w^f \in]\mathcal{W}_A^f(a), \mathcal{W}_{\underline{C}}^f(a)]$, in Regime VIII when $w^f \in]\mathcal{W}_{\underline{C}}^f(a), \mathcal{W}_D^f(a)]$, in Regime VI when $w^f \in]\mathcal{W}_D^f(a), \mathcal{W}_H^f(a)]$ and in Regime IX again when $w^f > \mathcal{W}_H^f(a)$.

Step 2.2: Cases where $a \geq A_2 = c_m$

For $a > c_{\mathrm{m}}$, Regime VII no longer exists: a couple can procreate for any $w^f \geq 0$. Regimes X and XI become reachable. Note that as long as $w^f < \mathcal{W}_D^f(a)$, only Regimes IX, VIII and X can be reached.

Case 4: $a \in]A_2, A_3[$

Now $\mathcal{W}_G^f(a) < 0$. As long as $w^f < \mathcal{W}_D^f(a)$, only Regimes IX, VIII and X can be reached. We now have to compare Regimes VIII and IX for $w^f < \mathcal{W}_D^f(a)$. As $a > A_0$, $U(c^f_{\mathrm{VIII}}, c^m_{\mathrm{VIII}}, n_{\mathrm{VIII}}) > U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, 0)$ for $w^f \in [\mathcal{W}_{\underline{C}}^f, \mathcal{W}_{\overline{C}}^f]$. This implies that Regime IX prevails $\forall\ w^f \in [0, \mathcal{W}_{\underline{C}}^f[$.

As shown above, if $a > A_1$, then $\mathcal{W}_D^f(a) \in [\min\{\mathcal{W}_{\underline{C}}^f, \mathcal{W}_{\underline{H}}^f\}, \max\{\mathcal{W}_{\overline{C}}^f, \mathcal{W}_{\overline{H}}^f\}]$. This implies that Regime VIII prevails $\forall w^f \in [\mathcal{W}_{\underline{C}}^f, \mathcal{W}_D^f[$ while Regime VI prevails $\forall w^f \in [\mathcal{W}_D^f, \mathcal{W}_{\overline{H}}^f[$. Finally, Regime IX prevails when $w^f \geq \mathcal{W}_{\overline{H}}^f$.

Case 5: $a \in]A_3, A_4[$

In line with previous cases and Step 1, under Assumption 2, we know that, for $a \in]A_3, A_4[$ $\mathcal{W}_G^f < \max\{\mathcal{W}_{\underline{C}}^f(a), \mathcal{W}_E^f(a)\} < \mathcal{W}_D^f(a) < \mathcal{W}_{\overline{H}}^f(a)$. We can conclude that Regime X prevails when $w^f < \mathcal{W}_G^f(a)$ while Regime IX prevails when $w^f \in]\mathcal{W}_G^f(a), \max\{\mathcal{W}_{\underline{C}}^f(a), \mathcal{W}_E^f(a)\}[$. When $w^f \in [\max\{\mathcal{W}_{\underline{C}}^f(a), \mathcal{W}_E^f(a)\}, \mathcal{W}_D^f(a)[$, Regime VIII is chosen by the couple while Regime VI is preferred to others when $w^f \in [\mathcal{W}_D^f(a), \mathcal{W}_{\overline{H}}^f(a)]$. Finally, for $w^f > \mathcal{W}_{\overline{H}}^f(a)$, Regime IX prevails again.

Case 6: $a \in]A_4, A_5[$

The only difference between Case 6 and Case 5 lies in the fact that for $a \in [A_4, A_5[$, $\mathcal{W}^f_G > \mathcal{W}^f_E(a)$. Therefore, we have that $\mathcal{W}^f_E < \mathcal{W}^f_{\underline{C}}(a) < \mathcal{W}^f_D(a) < \mathcal{W}^f_{\overline{H}}(a)$. The results of Case 6 directly follow.

<u>**Case 7:** $a > A_5$</u>

When $a > A_5$, the eat and procreate regimes no longer exist. Indeed, a woman with a wage equal to zero (or equivalently, having the maximum number of children) would consume more than c_{m}. This implies that only Regimes VI, IX and XI potentially exist. Under Assumption 2, $\mathcal{W}^f_F(a) < \mathcal{W}^f_I(a) < \mathcal{W}^f_{\overline{H}}(a)$. For $w^f < \mathcal{W}^f_F(a)$, $U(c^f_{\mathrm{XI}}, c^m_{\mathrm{XI}}, n_{\mathrm{XI}}) > U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, n_{\mathrm{IX}})$.

By continuity, we know that for $w^f = \mathcal{W}^f_F(a)$, $U(c^f_{\mathrm{VI}}, c^m_{\mathrm{VI}}, n_{\mathrm{VI}}) > U(c^f_{\mathrm{IX}}, c^m_{\mathrm{IX}}, n_{\mathrm{IX}})$ (which is why we know that $\mathcal{W}^f_F(a) < \mathcal{W}^f_{\overline{H}}(a)$). This implies that for $w^f \in [\mathcal{W}^f_F(a), \mathcal{W}^f_{\overline{H}}(a)]$, Regime VI prevails, while for $w^f > \mathcal{W}^f_{\overline{H}}(a)$, Regime IX prevails.

■

C Simulation and Robustness

C.1 Computation of the Standard Errors

Two methods are used in the literature to obtain the standard errors of parameters estimated by the simulated method of moments: bootstrapping and the delta method. The static nature of the model makes the bootstrapping method preferable to the delta method which is usually faster but tends to underestimate the standard errors. We first drew 200 random new samples with replacement from the original data. The new bootstrap samples were the same size as the original one (1,127,080 observations) but the frequency of each observation changed. For each of these new datasets we generated the 48 moments and estimated the corresponding parameters. We then computed the standard error of these estimators. By doing so, the uncertainty surrounding our estimated parameters cames exclusively from the uncertainty around the estimated moments. The results are reported in Column (2) of Table 11, and in Table 2 of the main text.

We expected the uncertainty coming from the randomness of the artificial population used to simulate the model to be minimal with a large enough T (typically $T = 100,000$ per education category). To check this expectation we also provide in Column (1) of Table 11 the parameters when the only uncertainty comes from the model: that is when we estimate the parameters 200 times using the same empirical moments but drawing different households

	Model only (1)		Data only (2)		Both (3)	
	mean	s.e.	mean	s.e.	mean	s.e.
σ_a	0.312	0.0087	0.312	0.0098	0.313	0.0098
m_a	0.849	0.0096	0.848	0.0109	0.849	0.0113
ν	6.687	0.1138	6.683	0.1090	6.674	0.1238
c_{m}	0.311	0.0036	0.311	0.0046	0.311	0.0048
μ	0.328	0.0065	0.328	0.0066	0.328	0.0076
$\underline{\theta}$	0.574	0.0066	0.574	0.0082	0.574	0.0082
α	0.596	0.0033	0.597	0.0041	0.597	0.0040
ϕ	0.224	0.0024	0.224	0.0022	0.225	0.0025
η	0.201	0.0077	0.201	0.0081	0.201	0.0087

Table 11: Mean and Standard Errors of Parameters

from the distribution. Column (3) of Table 11 presents the parameters when we combine both uncertainties, using the empirical moments from the bootstrap samples and drawing different households from the distribution. The difference between Columns (2) and (3) is very small.

C.2 Identified Parameters for Subsamples

Identification by Races. In the main text we have assumed an homogeneous marriage market. Here we assume instead that there are fragmented markets for each race separately (see Appendix A.6 for how the groups are constructed). We therefore reestimate the parameters for each race independently. The results are provided in Table 12.

Black fathers are characterized by a lower involvement in childrearing and a lower fixed cost of having children. Non-labor income is less dispersed for Whites and more dispersed for Hispanics.

Identification Removing Disabled. Since 83.5% of single childless women with no schooling are disabled, we also identify the parameters of the model after removing the disabled from the data. We can check that the c_{m} parameter still plays a role, even though its estimated value is lower. The results without the disabled are shown in Table 13.

Parameter	All	Blacks	Whites	Natives	Asians	Hispanics
	(1,127,080)	(71,022)	(967,315)	(6,051)	(26,193)	(56,064)
σ_a	0.312	0.320	0.198	0.203	0.200	0.488
m_a	0.848	0.868	0.914	0.950	0.896	0.865
ν	6.683	6.857	7.973	7.423	7.458	7.511
c_{m}	0.311	0.334	0.404	0.327	0.334	0.275
μ	0.328	0.395	0.169	0.363	0.321	0.242
$\underline{\theta}$	0.574	0.429	0.750	0.551	0.541	0.586
α	0.597	0.835	0.391	0.676	0.628	0.656
ϕ	0.224	0.196	0.242	0.206	0.219	0.214
η	0.201	0.116	0.159	0.177	0.201	0.117

Table 12: Identified Parameters by Race

Parameter	"All"	Without "disabled"
σ_a	0.312	0.286*
m_a	0.848	0.867
ν	6.683	6.708
c_{m}	0.311	0.263*
μ	0.328	0.320
$\underline{\theta}$	0.574	0.543*
α	0.597	0.577*
ϕ	0.224	0.227
η	0.201	0.205

* indicates a significant difference from "all".

Table 13: Identified Parameters Without the "Disabled"

C.3 Changes in the Time Allocation Parameter α

In our model we have assumed that the share mothers (α) and fathers ($1-\alpha$) devote to childrearing is constant across education groups and over time. Bianchi et al. (2004) show that the ratio of married mothers' to married fathers' time in child care declined between the mid-1960s and the late 1990s, which could be an indication that either social norms changed for all education categories, or that the increase in the education of women makes it optimal for fathers to spend more time with their children.

To have some insights on the role of α we carried out the following two experiments. First, fixing $\alpha = 1$ instead of identifying it from the cross sectional data, and re-estimating the rest of the parameters, we find that the quality of the match is lower: the model becomes unable to reproduce (a) a reasonable marriage rate (especially for highly educated women who have lost their incentive to marry), (b) childlessness rates for highly educated married women (for whom the cost of raising children becomes extremely high so they have more incentives to be voluntarily childless), (c) the gap between the fertility of married mothers and that of single mothers, who now face the same opportunity cost. Hence, allowing $\alpha < 1$ is important to generate the nice features of the model.

Second, consider that couples set α optimally under the constraint $\alpha \in [1/2, 1]$. A full-fledged model with proper bargaining on α would be the topic of another paper; however, a simple benchmark would be to assume the following rule:

$$\alpha = \begin{cases} 1 & \text{if } w^f < w^m \\ 1/2 & \text{if } w^f \geq w^m. \end{cases}$$

With this specification the marriage rates are reasonable, the U-shaped relationship between childlessness and education of married women is preserved, but the model fails in reproducing the high fertility of poorly educated married mothers (by about one child for the two lowest education categories), as poor married mothers face almost the same incentives as poor single mothers. Otherwise, the simple *ad-hoc* rule above does rather well, which indicates that bargaining over α could be a promising extension. Finally, we also tried to make α depend on the education of the mother, independently of the education of the father: $\alpha = 1 - w^f/2$. The properties of this simulation are similar to those in the previous case.

C.4 Fit of the Completed Fertility for Husbands

We compare here the simulated fertility of husbands to data.

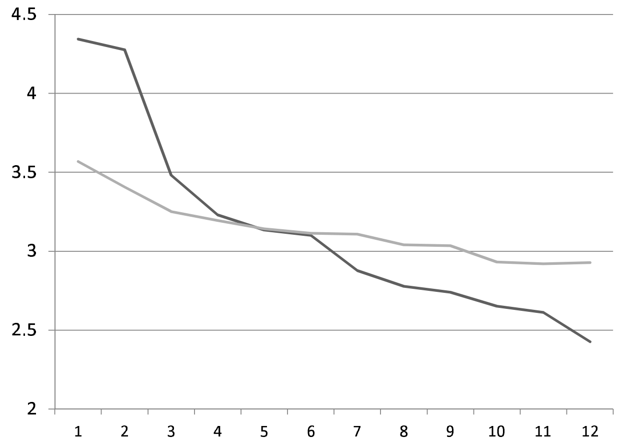

Figure 19: Completed Fertility of Husbands, by Education Categories. Data (black) and Simulation (grey)

C.5 Assortative Matching

The number of marriages by education category are given in Table 14. This table is constructed in the following way: first, we drop from the data all the individuals who are not married (MARST > 1) or who do not have an identified partner in the Census (SPLOC$= 0$). Then, we sort observations, first, by their serial number, corresponding to the household, and then by their sex, so that the man of the household comes before his wife in the data. We then generate a variable saying that the husband has a corresponding wife after him (the serial number for both has to be the same). The last step is to generate a variable with the education of the husband and another variable with the education of the wife.

The ratio

$$\frac{z(i,j)}{\sum_j z(i,j)}$$

gives the proportion of men of type j having married a woman of type i. Dividing this number by the proportion of women of type i in the total population,

$$\frac{\sum_i z(i,j)}{\sum_{i,j} z(i,j)},$$

we obtain the data in Table 15. Each cell gives, for each married man, his increased chances marrying a woman in a given category of education, compared with a purely random matching framework. If there was no assortativeness, all the cells would equal 1. The first cell means that a man of Category 1 has 57 times more chances of marrying a woman of education Category 1 than in the case of pure random matching. We are able to compute similar statistics with the simulated data, at the bottom of the table. In the simulation we also have some assortative matching, although lower than in the data. Regressing each cell in the top part of the table on the corresponding cell in the bottom part leads to an R^2 of 0.217 meaning that we are able to account for 21.7% of the variations in the assortativeness matching.

To assess the influence of having abstracted from exogenous assortative matching on the estimated parameters, we conducted the following exercise. Assume that a share λ of the population draws a potential spouse from his/her education category. The remaining share $1 - \lambda$ draws his/her partner randomly from the whole population. For different values of λ, we reestimate the parameters, minimizing the same objective as before. Table 16 presents the results. The case $\lambda = 0$ refers to the benchmark case of Table 2.

Most parameters are not much affected by the introduction of assortative matching. The parameter which seems to be the most sensitive to λ is α, the share of wives involved in childrearing. The simulated moments are also quite insensitive to the choice of λ. The value of λ which minimizes the objective function is around 0.3 - 0.4 (remember that we did not try to fit marriage data in the objective function). For $\lambda = 0.3$, we obtain the marriage matrix presented in Table 17. We conclude that introducing some exogenous assortative matching, captured by $\lambda > 0$, allows us to get much closer to the observed assortativeness, without modifying the estimated parameters or the ability of the model to reproduce the targeted empirical moments very much.

	1	2	3	4	5	6	7	8	9	10	11	12
1	71,179	13,086	17,869	3,842	2,971	1,751	13,915	4,122	1,630	1,431	988	332
2	14,465	68,397	53,512	7,801	5,114	3,638	18,566	5,054	1,398	1,543	778	104
3	26,113	98,004	472,453	79,705	72,697	43,995	229,332	58,307	14,805	14,369	7,674	1,004
4	6,114	23,070	135,612	88,745	53,731	34,170	151,668	38,420	7,745	8,634	3,924	560
5	6,353	22,146	159,403	69,339	132,115	62,352	282,405	75,275	16,301	18,667	7,760	1,026
6	4,745	15,159	125,973	58,541	83,086	94,623	256,444	73,902	14,284	17,530	7,061	1,066
7	23,125	49,898	565,508	264,819	380,715	300,185	3,455,022	1,261,526	283,977	595,232	251,575	28,998
8	4,840	8,097	92,315	48,220	71,447	57,596	672,543	837,035	159,921	525,637	300,613	44,805
9	1,337	2,726	23,806	12,216	17,571	15,488	186,588	169,093	98,724	188,244	125,071	16,869
10	1,245	2,095	17,462	8,040	12,546	11,090	170,890	209,703	56,760	548,646	445,586	86,506
11	916	1,519	11,343	5,403	8,345	7,326	96,804	105,051	28,313	180,051	292,392	87,314
12	146	63	575	231	374	234	3,409	3,883	1,541	9,598	17,766	22,918

Table 14: Marriages per Education Category. Men in Columns, Women in Rows.

	Men education category											
	1	2	3	4	5	6	7	8	9	10	11	12
Data												
1	57.233	5.553	1.377	0.767	0.456	0.357	0.324	0.187	0.307	0.088	0.087	0.147
2	8.584	21.421	3.043	1.149	0.580	0.548	0.319	0.169	0.194	0.070	0.051	0.034
3	2.499	4.950	4.332	1.893	1.329	1.069	0.636	0.315	0.332	0.105	0.081	0.053
4	1.185	2.359	2.518	4.268	1.989	1.681	0.852	0.421	0.352	0.127	0.084	0.060
5	0.797	1.466	1.916	2.159	3.166	1.986	1.027	0.534	0.479	0.178	0.107	0.071
6	0.675	1.138	1.717	2.067	2.258	3.418	1.058	0.594	0.476	0.190	0.110	0.084
7	0.332	0.378	0.777	0.943	1.043	1.093	1.437	1.023	0.955	0.650	0.397	0.229
8	0.184	0.162	0.335	0.454	0.517	0.554	0.739	1.794	1.421	1.517	1.253	0.936
9	0.167	0.180	0.285	0.378	0.419	0.491	0.675	1.192	2.886	1.788	1.715	1.160
10	0.085	0.075	0.114	0.136	0.163	0.192	0.338	0.808	0.906	2.846	3.337	3.248
11	0.119	0.104	0.141	0.174	0.207	0.241	0.364	0.770	0.861	1.779	4.170	6.242
12	0.257	0.059	0.097	0.101	0.126	0.105	0.174	0.387	0.636	1.287	3.441	22.248
Simulation												
1	1.165	1.116	1.079	1.060	1.044	1.006	0.993	0.981	0.969	0.961	0.960	0.952
2	1.127	1.087	1.059	1.042	1.027	1.009	1.001	0.990	0.978	0.966	0.954	0.909
3	1.100	1.061	1.044	1.034	1.028	1.016	1.007	0.999	0.992	0.964	0.938	0.893
4	1.074	1.046	1.037	1.022	1.019	1.013	1.005	1.000	0.996	0.972	0.954	0.918
5	1.055	1.041	1.028	1.015	1.015	1.011	1.004	0.999	0.996	0.977	0.966	0.935
6	1.038	1.031	1.020	1.010	1.009	1.007	1.002	0.999	0.997	0.985	0.978	0.954
7	1.014	1.016	1.011	1.006	1.004	1.005	1.000	0.999	1.000	0.994	0.992	0.975
8	0.987	0.993	0.996	0.996	0.999	1.000	0.999	1.000	1.001	1.002	1.009	1.008
9	0.961	0.972	0.982	0.988	0.994	0.995	0.996	1.000	1.003	1.014	1.026	1.038
10	0.892	0.912	0.939	0.966	0.974	0.979	0.991	1.007	1.010	1.043	1.067	1.149
11	0.834	0.867	0.909	0.946	0.954	0.967	0.988	1.009	1.021	1.066	1.100	1.208
12	0.669	0.740	0.792	0.845	0.880	0.909	0.957	1.008	1.061	1.180	1.263	1.557

Table 15: Assortative Matching

λ	0	0.1	0.2	0.3	0.4	0.5
$f(p)$	45.905	45.220	42.268	41.243	40.669	54.786
σ_a	0.312	0.298	0.292	0.294	0.297	0.289
m_a	0.848	0.851	0.846	0.892*	0.882*	0.822*
ν	6.683	6.585	6.719	6.405*	6.324*	6.499
c_{m}	0.311	0.303	0.291*	0.278*	0.233*	0.287*
μ	0.328	0.330	0.332	0.323	0.326	0.327
$\underline{\theta}$	0.574	0.583	0.578	0.613*	0.635*	0.637*
α	0.597	0.584*	0.546*	0.497*	0.461*	0.616*
ϕ	0.224	0.224	0.223	0.236*	0.236*	0.224
η	0.201	0.217	0.209	0.221*	0.223*	0.230*

* indicates a significant difference from the case $\lambda = 0$.

Table 16: Identified Parameters for λ between 0 and 0.5

	Men's education category											
	1	2	3	4	5	6	7	8	9	10	11	12
1	27.963	0.746	0.731	0.708	0.703	0.692	0.683	0.670	0.657	0.594	0.555	0.456
2	0.753	18.376	0.736	0.710	0.709	0.702	0.692	0.678	0.66	0.599	0.562	0.463
3	0.735	0.735	4.044	0.710	0.711	0.705	0.694	0.686	0.673	0.624	0.590	0.495
4	0.722	0.724	0.729	9.304	0.707	0.701	0.697	0.692	0.689	0.645	0.614	0.524
5	0.712	0.721	0.726	0.705	7.066	0.703	0.698	0.694	0.691	0.654	0.627	0.542
6	0.700	0.710	0.721	0.701	0.706	9.031	0.699	0.697	0.694	0.667	0.643	0.566
7	0.691	0.700	0.715	0.698	0.702	0.700	1.657	0.699	0.700	0.678	0.659	0.590
8	0.682	0.689	0.709	0.691	0.697	0.697	0.699	2.503	0.705	0.691	0.677	0.625
9	0.656	0.679	0.697	0.682	0.691	0.693	0.698	0.703	7.709	0.703	0.696	0.667
10	0.622	0.640	0.663	0.663	0.678	0.685	0.693	0.705	0.72	3.256	0.736	0.743
11	0.605	0.605	0.641	0.646	0.665	0.676	0.688	0.701	0.721	0.742	4.452	0.802
12	0.479	0.501	0.537	0.566	0.582	0.612	0.640	0.672	0.718	0.787	0.829	23.491

Table 17: Simulated Marriage Matrix for $\lambda = 0.3$

C.6 Increase in Inequality

Here we draw the evolution of simulated childlessness rates for alternative levels of inequality.

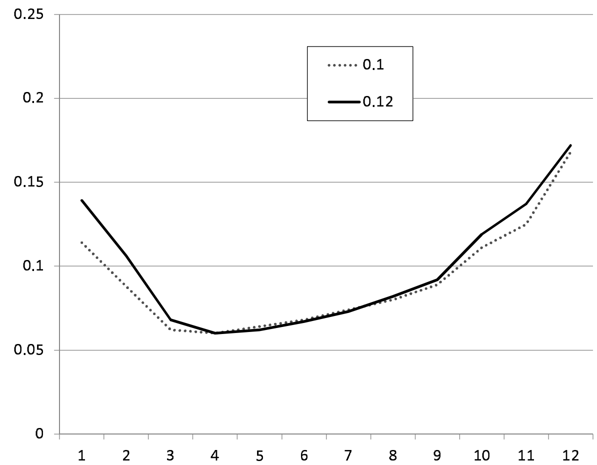

Figure 20: Childlessness Rate for Different Levels of Inequality (Mincer Coefficient) - Married

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