# Keeping-up with the Joneses, a new source of endogenous fluctuations

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# Document de travail

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# Keeping-up with the Joneses, a new source of endogenous fluctuations

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**Abstract :** *Our main objective is to study the impact of consumption externality like keeping of with the Joneses on the properties of long-run equilibrium in the two-sector optimal growth model. Does this consumption externality lead to a new mechanism of local indeterminacy and endogenous fluctuations ? We will see that, in two-sector growth models with exogenous labor and without technological externalities, if the representative agent is able to give more value to his social status than his own consumption, this is the keys of a new mechanism for endogenous fluctuations. Moreover, by opposition with the other endogenous fluctuation mechanisms, we will see that this one doesn't need to have restriction on the factor intensity configuration of the consumption sector.*

**Keywords :** *Two-sector models, continous-time models, consumption externality, keeping up with the Joneses, local indeterminacy, endogenous fluctuations.*

# 1 Introduction

We propose an extension of the papers of Garnier, Venditti and Nishimura [8, 9] where endogenous fluctuations can be obtain without both technological externalities and restrictions on the capital intensity configuration of the consumption sector. Local indeterminancy is obtained only by adding a consumption externality in preferences (i.e. keeping up with the Joneses). This one adds an exogenous variable that represents the consumption standard of the economy. To obtain this new endogenous fluctuations mechanim, we assume that the marginal utility of consumption is increasing with respect to the externalities. Recall that the intertemporal substitution elasticity of the consumption (IESC) measures the responsiveness of the consumption growth to the interest growth rate but this consumption externality implies the existence of two values of the IESC :

• The first, called IESC at the private level, doen't integrate the externality and is positive. I note this elasticity by $\epsilon_{cc}$in the paper. It measures responsiveness of the consumption growth to the interest growth rate of the representative agent.

• The second, called IESC at the social level (i.e. evaluted at the symetric equilibrium), integrates externality and can be negative here since we assume that agents can give more weight to their social status rather than to their own present consumption. IESC at the social level measures the responsiveness of the consumption growth to the interest growth rate of all society. I note this elasticity by $\alpha$ in the paper. Consequently, this consumption externalty breaks the symetry of the responsiveness of the consumption growth to the interest growth rate between the private and social levels.

This idea is of the same kind of the one used in the models where technological externalities break the duality between price and quantity effects

# 2 The Model

We consider an infinite horizon, continous-time, two-sector model with Cobb-Douglas technologies, inelastic labor supply and consumption externalities through "keeping up with the Joneses" preferences. The economy consists of competitive firms and a representative household.

### 2.1 Firms

The consumption good $y_0$ and capital good $y_1$ are produced by capital $x_{1j}$ and labor $x_{0j}$, $j = 0, 1$, through a Cobb-Douglas technology but without sector-specific externalities and with constant returns to scale : see the paper Garnier *et al* [9] p. 328 equation (2) with $\beta_{ij} = \hat{\beta}_{ij}$, so all the technological structure of the paper holds.

### 2.2 Household

We assume that the population is constant and normalized to one and the labor suply is inelastic i.e. according to the case where the labor supply elasticity is infinite in the paper of Garnier *et al* [9]. The representative agent derives his utility from consumption $c(t)$ and faces consumption externality $z(t)$, that is : $U\left(c(t), z\left(t\right)\right)$. The consumption externality is given by the consumption average of the economy which can be interpreted as the consumption standard. The function $U$ satisfies the standard hypothesies but the introduction of consumption standard implies that consumption spillovers affect the household's utility. The "keeping up with the Joneses" formulation of preference is such that the marginal utility of consumption rises with the consumption standard. Hence the following assumption holds :

**Assumption 1** *: $U(., z(t))$ is increasing and concave $\forall z(t) \in \mathbb{R}^{+}$, the first partial derivatives satisfy $U_z(c(t), z(t)) > 0$ and $U_c(c(t), z(t)) > 0$ and the second partial derivatives satisfy $U_{cz}(c(t), z(t)) > 0$ and $U_{cc}(c(t), z(t)) < 0$*

We introduce the following elasticities :

$$\epsilon_{cc} = -\frac{U_c\left(c(t), z\left(t\right)\right)}{c(t)U_{cc}\left(c(t), z\left(t\right)\right)} > 0 \quad (1)$$

$$\epsilon_{cz} = \frac{U_c\left(c(t), z\left(t\right)\right)}{c(t)U_{cz}\left(c(t), z\left(t\right)\right)} > 0 \quad (2)$$

where $\epsilon_{cc}$ is the private Intertemporal Elasticity of Susbstitution in Consumption (IESC) and $\epsilon_{cz}$ is the elasticity of substitution between consumption and consumption standard.

The enhanced value of the consumption standard by the agent is positively correlate to $\epsilon_{cz}$. We introduce the ratio $\frac{c(t)}{z(t)}$ as the social status of the agent.

We can have $\epsilon_{cc} < \epsilon_{cz}$ i.e. $U_{cc} + U_{cz} > 0$ that is, we consider that the agents may give more weight to their social status rather than to their own present consumption.

The objective of the representative agent is to solve the following intertemporal optimization problem by taking $z(t)$ as given :

$$\begin{array}{ll} \max\limits_{y_1(t),x_1(t)} & \int_0^\infty e^{-\delta t} U\left(c(t), z(t)\right) dt \\ \text{s.c.} & \dot{x}(t) = y_1(t) - g x_1(t) \\ & x_1(0) = x_1 \;\; \text{given} \end{array} \tag{3}$$

## 2.3 The competitive equilibrium

Let us denote by $\alpha$ the social IESC :

$$\alpha = \frac{1}{\epsilon_{cc}} - \frac{1}{\epsilon_{cz}} \tag{4}$$

If $\epsilon_{cc} < (>)\ \epsilon_{cz}$ then we have $\alpha > (<) 0$. We note that $\alpha$ plays the same role that the parameter $\sigma$ in the dynamical system (13) of the model of Garnier *et al [8]* p.331, but here $\alpha$ can also be negative (i.e. $\alpha < 0$) that corresponds to the complementary part of the half-line $\triangle_\infty$ describes in the paper p.244 section 3.3 (i.e inelastic labor supply) but without sector-specific externalities. Therefore both dynamical system and steady-state $(x_1^*, p_1^*)$ are the same than the ones of Garnier *et al [9]* p.331 equation (13).

In order to study the indeterminacy properties of equilibrium, we linearize the dynamical system around $(x_1^*, p_1^*)$ which gives a Jacobian characterized by a Trace and a Determinant.

In this type of models the steady state is locally indeterminate if and only if the Jacobian matrix has two eigenvalues with negative real part what is equivalent to verify that $T<0$ et $D>0$. Moreover, Trace and Determinant are functions of $\alpha$, then when $\alpha$ gets from $-\infty$ to 0, $T(\alpha)$ and $D(\alpha)$ move along the line called in what follows $\Delta_\alpha$(see Garnier *et al [9]*).

## 3 Endogenous fluctuations and social status

Here, the case $\alpha>0$ doesn't interest us since it cannot provide local indeterminacy : in this case, Garnier, Venditti and Nishimura [7] have shown that sector-specific externalities are needed to have local indeterminacy. Consequently, we only focus on the case where the level of consumption externality is sufficiently large to have $\epsilon_{cc}>\epsilon_{cz}$ and thus $\alpha<0$ i.e. agent favours his social status $\frac{c(t)}{z(t)}$ more than his own consumption $c(t)$ over time.

We study the variations of $T(\alpha)$ and $D(\alpha)$ in the $(T,D)$ plane, when $\alpha$ varies continuously on $]-\infty,0[$.

So, when $\alpha$ gets from $-\infty$ to 0, the pair $(T(\alpha),D(\alpha))$ moves along the half line $\Delta_\alpha$ [1] characterized by a starting point $(T(\infty),D(\infty))$ :

$$T(\infty) = -\frac{\frac{dc}{dx_1}\frac{dy_1}{dp_1}}{\frac{dc}{dp_1}}+\frac{dy_1}{dx_1}-g \tag{5}$$

$$D(\infty) = 0 \tag{6}$$

and a ending point $(T(0),D(0))$ such that :

$$D(0) = \left(\frac{dy_1}{dx_1}-g\right)\left(\delta+g-\frac{dw_1}{dp_1}\right)<0 \tag{7}$$

$$T(0) = \delta \tag{8}$$

Consequently, we know from benhabib and Nishimura [5] that, without sector-specific externalities, indeterminacy is ruled out for $\alpha=0$ (i.e. ending point) whereas the starting point is located on the abscissa area since $D(\infty)=0$ . We have to locate both starting and ending points to draw the line and to show that it gets in the local indeterminacy area.

1. When $\alpha>0$, the pair $(T(\alpha),D(\alpha))$ moves along the another part of the half line $\Delta_\alpha$ : see Garnier, Venditti and Nishimura [8].

We can verify both $T(\infty) < 0$ and $D$ is an increasing function of $\alpha$, then the half line $\Delta_\alpha$ gets by the indeterminacy area.

More precisely, there exists $\bar{\alpha} \in \left]-\infty, 0\right[$ such that the steady state is locally indeterminate $\forall \alpha \in \left]-\infty, \bar{\alpha}\right[$.

Hence, we give the following proposition :

**Lemma 1** *: Under assumptions 1 then :*

$\exists \overline{\alpha} \equiv \frac{1}{\frac{p_1}{c}\frac{dc}{dp_1}} \in \left]-\infty, 0\right[$ *such that* $\forall \alpha < \overline{\alpha}$ *the steady state* $(x_1^*, p_1^*)$ *is locally indeterminate and* $\forall \alpha > \overline{\alpha}$ *the steady state* $(x_1^*, p_1^*)$ *is saddle point.*

(See the proof in the appendix).

This lemma gives a condition set leading endogenous fluctuations without restrictions on the factor intensity of the consumption good.

Does it possible to have a negative IESC at the social level and how can we interpret it ? This possibility has been yet studied by Robert Hall [11]. He explains that a "detailed study of data for the twentieth-century United States Shows no strong evidence that the elasticity of intertemporal substitution is positive and that earlier finding of substantially positive elasticities are reversed when appropriate estimation methods are used". Even if a lot of empirical estimations of IESC give a really small positive value there is some extimations that give a negative value (Hansen and Singleton 1988)

## 3.1 Endogenous fluctuations mechanism

Now, starting from an arbitrary equilibrium, consider that he expects another one with a higher rate of investment and higher level of capital stock coming from an instantaneous increase in relative price of investment good $p_1$. The only way that this other equilibrium path becomes a new equilibrium is to find a mechanism which reverses the price toward the equilibrium and offsets this initial increase. Suppose, for example, that the investment good is capital intensive, then, from the Rycbzynsky theorem, a higher capital stock, at constant prices, implies an increase (more than proportional) of output of the investment sector and a decrease (more than proportional) of output of the consumption sector : the consumption social standard decreases. Now from Stolper-Samuelson theorem, the initial price rise leads to an increase in the rate of return of capital, given by $w_1$, and to maintain the equality of the overall return to capital and discount rate : $\frac{\dot{p_1}}{p_1}+\frac{w_1(p_1)}{p_1}-\alpha\left[\varepsilon_{cp}\frac{\dot{p_1}}{p_1}+\varepsilon_{cx}\frac{\dot{x_1}}{x_1}\right]=\delta+g$ [2], the price of the investment good must decline. However, this is not enough to check the rycbzynsky effect. But there is another effect coming from IESC at the social level ($\alpha<0$) : there are two opposite effects playing through the decreasing marginal utility in consumption (i.e. $U_{cc}<0$) and through the positive effect of externality on this marginal utility (i.e. $U_{cz}>0$ ) but the last is higher and the marginal utility decreases. Consequently, the loss in the present consumption will be not offset in the future, that is, a decrease in the social stantard implies a future decrease of the consumption level.

The IESC measures the responsiveness of the consumption growth to the interest rate : let ‘s consider a present decreasing of the interest rate such that the aggregate consumption decreases for one unity, thus the social status decreases for one unity. Normally, this decreasing has to be offset in the future. But here, the representative agents want to keep their social status consequently if the present social status decreases for one unity agents decrease their consumption in order to keep it constant over time. So, all agents wants only to keep their social status and they don’t have to postpone the present decreases of consumption. That is measured by the IESC at the aggregate level : a present decrease of the social status implies a future decreases of the consumption.

2. With $\varepsilon_{cp}=\frac{p_1}{c}\frac{\partial c}{\partial p_1}$ the price-elasticity of the consumption and $\varepsilon_{cx}=\frac{x_1}{c}\frac{\partial c}{\partial x_1}$ the capital-elasticity of the consumption and they are negatives.

## 4 Examples

Let us consider this utility function :

$$U(c(t), z(t)) = \frac{1}{1-\sigma}\left[c(t)^{1+\gamma}\left(\frac{c(t)}{z(t)}\right)^{-\gamma}\right]^{1-\sigma}$$

where $1 > \gamma > 0$ a measure of the weight of the social status $c/z$ in preferences and $\sigma > 0$ the individual risk aversion or the intertemporal elasticity of substitution in consumption at the private level. This function satisfies assumption 1 and :

$$\begin{aligned} \epsilon^*_{cc} &= \frac{1}{\sigma} \\ \epsilon^*_{cz} &= -\frac{1}{\gamma(1-\sigma)} \end{aligned}$$

Therefore :

$$\alpha = \sigma - \gamma(1-\sigma)$$

Note that for any $1 > \gamma > 0$, if $\sigma = 0$ (i.e. the utility is linear with respect to the consumption) then $\alpha = -\gamma < 0$ and if $\sigma = 1$ then $\alpha = 1$ Therefore $\forall \gamma \in ]0, 1[$, $\exists \tilde{\sigma} \in [0, 1[$ such that $\forall \sigma \in [0, \tilde{\sigma}[$ we have $\alpha < 0$.

We want to illustrate the lemma 1 for both capital and labor intensive consumption good.

We use a parametrization according empirical results on the labor share in european countries :

$$\beta_{01} = 0.62$$
$$g = 0.05$$
$$\delta = 0.01$$

If we set $\beta_{00} = 0.6$ the consumption good is capital intensive, moreover, we obtain the following critical value : $\overline{\alpha} = -0.05$. we can set for example, $\sigma = 0.2$ and $\gamma = 0.3125$. Otherwise, if we set $\beta_{00} = 0.65$ the consumption good is now labor intensive, the new critical value is $\overline{\alpha} = -0.01$, we can set, for example, $\sigma = 0.2$ and $\gamma = 0.2625$.

# 5 Concluding comments

We have shown that keeping up with the Joneses' externality in preferences are able to lead endogenous fluctuations of the steady-state without restriction on capital intensity configuration of sectors and without sector-specific externalities.

# 6 Appendix

## 6.1 Computation of derivatives used in $T(\alpha)$ and $D(\alpha)$

All derivatives corresponds to that given section A.3 p.347-351 in the paper of Garnier *et al* [9] without sector-specific externalities (i.e. $T = \hat{T}$) and with inelastic labor supply (i.e $\gamma \to \infty$).

## 6.2 Proof of the proposition 1

See Garnier, Venditti and Nishimura [9] for the proof of the computation of $\triangle_\infty$.

The computation of $\frac{dT}{d\alpha}$ and $\frac{dD}{d\alpha}$ give :

$$\frac{dT}{d\alpha} = \frac{p_1^*}{c^* E^2}\left[\frac{\partial c}{\partial x_1}\frac{\partial y_1}{\partial p_1} + \frac{\partial c}{\partial p_1}\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right)\right] \quad (9)$$

$$\frac{dD}{d\alpha} = \frac{p_1^*}{c^* E^2}\frac{\partial c}{\partial p_1}\left(\frac{\partial y_1}{\partial x_1} - g\right)\left(\delta + g - \frac{\partial w_1}{\partial p_1}\right) \quad (10)$$

Since $\frac{dc}{dp_1} = -\frac{1}{b^2}\left[(1-\beta_0)\, a_{11} + \beta_0 a_{01} x_1\right] < 0$ then $\frac{dD}{d\alpha} = \frac{p_1^*}{c^* E^2}\frac{dc}{dp_1} D\,(0) > 0$ and $T\,(\infty) < 0$.

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