Endogenous fluctutations: a financial transmission Mechanism

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Endogenous fluctutations: a financial transmission mechanism

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Abstract: *We propose to study the dynamic impact of adjustment costs in capital on the two sectors model with positive sector specific externalities. We proove that such costs are able to lead to endogenous fluctuations by financial transmission mechanism. Indeed, since adjustments costs are linked to the marginal Q of Tobin, the firm's investment decision depends on the gap between the true value of the assets of this firm and their market value. The marginal Q of Tobin is an indicator of this market value and when adjustment costs are sufficiently high they can interplay with sector-specific externalities to provide endogenous fluctuations. We can prove fluctuations and cycle arise for new configurations of capital intensity across sectors. Classically, in this model, these fluctuations take place with sufficiently high level of sector-specific externalities but only with capital intensity reversal across sector. When adjustment costs are considered, reversal is no longer necessary condition to endogenous fluctuations to arise. Moreover, we show that there exists a link between financial volatility, mesured by variations of the marginal Q of Tobin, and fluctuations.*

Keywords: *Two-sector growth model, externality, adjustment costs, endogenous fluctuations.*

JEL classification: C62, E32, 041

1 Introduction

The link between fluctuations of the real economic activities and the finance market volatility is clear but the study of this link in theoritical models is difficult. Indeed, how can we construct a deterministic model with realistic financial market? If we choose to think this link in the long-run, we may consider that by the canal of transmission of the investment we could explain indirectly the long-run movement of financial markets and more precisiely the price of assets. As financial market evaluates assets of the firms then volatility correponds to movments from periods where assets are undervaluated to periods where they are overvaluated. According the market value of its assets, a firm has to decide its investment plan, then a good indicator to take this decision is the gap between the true value of the asset of the firm and the market value, this indicator exists and it's called marginal Q of Tobin. Many studies have show that, in the long run the Q of Tobin (where the marginal Q of Tobin is evaluated by the average of Q of Tobin because of the difficulty to compute it) follows a dynamic near of the one of the financial index. Consequently, in the long run, we can use this marginal Q of Tobin to understand the way of moving of financial market. We propose to study a new way for endogenous fluctuations mechanism and its impacts on the financial market through the movment of the marginal Q of Tobin. This indicator appears with frictions on the transformation process of the investment in capital stock, that we call adjustment costs. The idea that the installation of a new capital could generate additional costs is widely viewed as an important feature of the investment decision analysis. Neoclassical studies of investment behavior often ignore variations in capacity utilization whereas lot of empirical models (DSGE) takes it account to fit the datas. As we will see, these costs lead to a gap between the true value of the assets of the firm and its market value what gives a value different from the unity to the Marginal Q of Tobin (indeed, without such costs, there is no gap and the marginal Q of Tobin is always equals to one). Consequently, as in the long-run, the correlation between the value of the marginal Q of Tobin and financial volatility is clearly established by econometric studies we think that adjustment costs allow us to make the link between endogenous fluctuations and volatility of the financial market .

Previous papers have already studied such costs but only in the one sector model. In this model, Jinill Kim [15] has only studied the interactions between the endogenous fluctuations mechanism coming from the presence of technological externalities and adjustment costs. Recall that in this model the presence of these externalities leads to increasing return to scale and that endogenous fluctuations depends on the level of these return to scale. Jinill Kim [15] proves these costs make it difficult for local indeterminacy to occur (i.e. necessary condition to have endogenous fluctuations in this type models). Indeed, the required degree of increasing returns is higher in the presence of such costs. Nevertheless, in the two sector model (with constant returns to scale at the social level and decreasing at the private level) the indeterminacy mechanism is quite different as it implies constant returns to scale. Benhabib and Nishimura [7] have proved that, with a separable utility function which is linear in consumption and strictly concave with respect to labor, local indeterminacy (i.e. endogenous fluctuations) arises if and only if technological externalities allow factor intensities reversal between private and social levels (i.e. the consumption good is capital intensive at the private level and labor intensive at the social level). Consequently, there is a technological mechanism arising from externalities which breaks the duality between the Rybczynsky and Stolper-Samuelson effects (i.e. price and quantity effects) and leads to endogenous fluctuations. Then, we can thing that the effects of adjustments costs in the two-sector model will be quite different than in the one sector version. We propose to examine the interaction of these costs with the existing mechanism of endogenous fluctuations proposed by Benhabib and Nishimura but we will show that the interplay between adjustment costs and sector-scpecific externalities can produce a new mechanism of endogenous fluctuations and cycles (.i.e. local indeterminacy and Höpf bifurcation) such that the capital intensity reversal is not required and where the consumption good is labor intensive what is in sharp contrast with all existing results in the litterature.

As I have explain beforer, I think these costs could allow us to make the long-run link beween economic fluctuations and financial volatility. Indeed, as adjustment costs lead to a gap between the real value of the asset of the firm and their market value then there exists a mechanism which plays through the investment decision of the firm and the value of the marginal Q of Tobin (used as decision variable by the firm). For example, if the marginal Q of Tobin is greater than unity then the actualised value of the market of the firm is greater than the real value of its assets then the firm is incited to invest more in capital. But the presence of sector specific externalities, the adjustment costs and the capital intensity configurations of each sector can impact negatively the outputs and the value of effective capital. All of that leads to a decrease of the market value of the firm and of the value of the marginal Q of Tobin (i.e. the the marginal Q of Tobin becomes lower than unity), consequently, the firm modifies his next investment decision that leads to a further fluctuations of the market value of its assets and of its outputs. The consequence is then endogenous fluctuations and financial volatility caused by the expectation of the market value of the firm that has to decide its investment plan.

In this way, we show that the presence of such costs make it possible indetermincay and Höpf bifurcation with a new configuration of externalities which doesn't lead to capital intensity reversal between the private and social level (with only positive sector specific externalities, exogenous labor and linear utility function in consumption) whereas it's not possible otherwise.

The rest of the paper is organized as follows. Section 2 describes the economy. Section 3 characterizes the competitive equilibrium. Section 4 analyzes the mechanism that leads to equilibrium indeterminacy. Section 5 gives an example of utility function that allows the existence of indeterminacy and illustrates our main result through a standard parametrization of the model. Section 6 concludes. All the proofs are collected in the appendix.

2 The economy

We consider an infinite horizon, continuous time, two-sector model with Cobb-Douglas technologies, inelastic labor supply and non linear utility function in consumption. The economy consists of competitive firms and a representative household.

2.1 Firms

We assume that consumption good y_0 and capital good y_1 are produced by capital x_{1j} and labor x_{0j}, $j = 0, 1$, through a Cobb-Douglas technology with sector-specific externalities e_j. The representative firm in each industry faces the following technology called *private production function*:

$$y_j = F_j\left(x_{0j}, x_{1j}\right) = x_{0j}^{\beta_{0j}} x_{1j}^{\beta_{1j}} e_j\left(\overline{X}_{0j}, \overline{X}_{1j}\right) \quad \text{for } j = 0, 1 \tag{1}$$

with $\beta_{ij} \in [0,1]$ and X_{ij} the average use of input i in the sector j.

The positive sector-specific externalities are such that:

$$e_j\left(\overline{X}_{0j}, \overline{X}_{1j}\right) = \overline{X}_{0j}^{b_{0j}} \overline{X}_{1j}^{b_{1j}} \tag{2}$$

We assume that this economy wide average are taken as given by each individual firms. At the equilibrium, since all firms of sector j are identical, we have $\overline{X}_{ij} = x_{ij}$ and we define the *social production function* as follows:

$$y_j = x_{0j}^{\hat{\beta}_{0j}} x_{1j}^{\hat{\beta}_{1j}} \quad \text{for } j = 0, 1 \tag{3}$$

with $\hat{\beta}_{ij} = \beta_{ij} + b_{ij}$. We assume that the returns to scale are constants at the social level and decreasing at the private level i.e. in each sector $j = 0, 1$, $\hat{\beta}_{0j} + \hat{\beta}_{1j} = 1$.

The labor is exogenous, therefore the total labor, normalized to one, is given by:

$$x_{00} + x_{01} = 1 \tag{4}$$

and the total stock of capital is given by x_1 such that:

$$x_{10} + x_{11} = x_1 \tag{5}$$

Choosing the price of consumption good as the numeraire, i.e. $p_0 = 1$, a firm in each industry maximizes its profit given the output price of the investment p_1, the rental rate of capital w_1 and the wage rate w_0. The first order conditions subject to the private technologies (1) give

$$x_{ij}/y_j = p_j \beta_{ij}/w_i \equiv a_{ij}(w_i, p_j), \quad i, j = 0, 1 \tag{6}$$

We call a_{ij} the input coefficients from the *private* viewpoint. If the agents take account of externalities as endogenous variables in profit maximization, the first order conditions subject to the social technologies (3) give on the contrary:

$$x_{ij}/y_j = p_j \hat{\beta}_{ij}/w_i \equiv \hat{a}_{ij}(w_i, p_j), \quad i, j = 0, 1 \tag{7}$$

We call $\hat{a}_{ij}$ the input coefficients from the *social* viewpoint. As we are showing below, the factor-price frontier, which gives a relationship between input prices and output prices, is expressed with the input coefficients from the social viewpoint. Based on these input coefficients it may be shown that the factor-price frontier is determined by the input coefficients from the social viewpoint while the factor market clearing equation depends on the input coefficients from the private perspective:*. Considering the external effects as given, profit maximization in both sector gives demand functions as function of capital stock $x_1(t)$, production level of the investment good $y_1(t)$ and external effects (e_0, e_1), namely $\tilde{x}_{ij} = x_{ij}(x_1, y_1, e_0, e_1)$ for $i, j = 0, 1$. The production frontier is then defined as:

$$\begin{array}{rcl} y_0 = T(x_1, y_1, e_0, e_1) & = & \underset{\tilde{x}_{ij}}{Max}\ \tilde{x}_{00}^{\beta_{00}} \tilde{x}_{10}^{\beta_{10}} e_0 \\ s.t. \quad (1) & (3) & (4) \end{array}$$

From the envelop we get: $\frac{\partial T}{\partial x_1} = w_1$ and $\frac{\partial T}{\partial y_1} = -p_1$.

Lemma 1 *: Denote $p = (1, p_1)'$, $w = (w_0, w_1)'$ and $\hat{A}(w, p) = [\hat{a}_{ij}(w_i, p_j)]$. Then $p = \hat{A}'(w, p)w$.*

and:

Lemma 2 *: Denote $x = (1, x_1)'$, $y = (y_0, y_1)'$ and $A(w, p) = [a_{ij}(w_i, p_j)]$. Then $A(w, p)y = x$.*

At the equilibrium, the rental rate is function of the output price only, i.e. $w_1 = w_1(p_1)$, while outputs are functions of the capital stock, total labor and the output price, $y_j = \tilde{y}_j(x_1, p_1)$, $j = 0, 1$.

2.2 Adjustment costs function and capital accumulation

We assume that the firm in the consumption sector only faces adjustment costs since it pays p_1 for each unity of investment good to the firm in the investment sector. But, because of adjustment costs in capital Φ when the firm decides to invest by paying p_1 for one unity of investment good y_1 that doesn't result in one more unity of capital, indeed†:

*See Garnier, Nishimura and Venditti [10] for the proofs of these results.

†We incorporate the investment adjustment costs in the capital accumulation equation in a similar way to Lucas and Prescott [16].

$$\dot{x}_1(t) = x_1(t)\Phi\left(\frac{y_1(t)}{x_1(t)}\right)$$

The adjustment costs could be thought as a mesure of the efficiency of the investment i.e. efficiency index of the investment.

For the later analysis of the local dynamics, we make assumptions on the specific form of the adjustment costs function [‡].

Assumption 1 *: Let g the depreciation rate of capital, the adjustment costs function satisfies:*

1. $\Phi(g) = 0$
2. $\Phi'(g) = 1$
3. $\Phi' > 0$ *and* $\Phi'' < 0$

The first assumption defines the depreciation rate, g, as the ratio between investment and capital at the steady state. The second assumption makes the steady state of our model with adjustment costs the same that the one with linear capital accumulation equation[§]. The third assumption ensures that the steady state is a maximum (second-order optimality conditions).

2.3 Household

The population is constant and normalized to one. At the date t, the representative agent derives his utility $U(.)$ from consumption $c(t)$. In this model, the representative agent consumes the whole consumption good, we have then: $c = y_0$ and he solves the following intertemporal maximization problem

[‡] We note that the classical expression of capital accumulation corresponds to the particular adjustment costs function $\Phi\left(\frac{y_1}{x_1}\right) = \frac{y_1}{x_1} - g$ with g the constant depreciation rate of capital.

[§] we add these costs such that only the dynamic around the steady-state is affecting but not the steady-state values

[¶]:

$$
\begin{array}{lll}
\max\limits_{y_1(t), x_1(t)} & \int_0^\infty e^{-\rho t} T(x_1(t), y_1(t), e_0(t), e_1(t)) dt & \\
\text{s.c.} & \dot{x}_1(t) = x_1(t)\Phi\left(\frac{y_1(t)}{x_1(t)}\right) & \\
 & x_1(0) = x_1 and\ \{e_0(t), e_1(t)\}_{t\geq 0} \qquad given &
\end{array} \tag{8}
$$

Where $\rho > 0$ is the subjective discount rate.
The Hamiltonian in current value of (8) is:

$$H = T\left(x_1(t), y_1(t), e_0(t), e_1(t)\right) + q_1(t)\left(x_1(t)\Phi\left(\frac{y_1(t)}{x_1(t)}\right)\right) \tag{9}$$

The first order conditions (F.O.C.) are:

$$p_1(t) = q_1(t)\Phi'\left(\frac{y_1(t)}{x_1(t)}\right) \tag{10}$$

$$\dot{q_1}(t) = q_1(t)\left[\rho - \Phi\left(\frac{y_1(t)}{x_1(t)}\right) + \frac{y_1(t)}{x_1(t)}\Phi'\left(\frac{y_1(t)}{x_1(t)}\right)\right] - w_1(t) \tag{11}$$

$$\dot{x_1}(t) = x_1(t)\Phi\left(\frac{y_1(t)}{x_1(t)}\right) \tag{12}$$

and the transversality condition:

$$\lim_{t\to+\infty} x_1(t)p_1(t)e^{-\rho t} = 0 \tag{13}$$

Where q_1 is the co-state variable which corresponds to the utility price of capital in current value.

2.4 Tobin's marginal Q

By definition the Tobin's marginal Q is given by the ratio of the shadow price of capital and the investment price: $q_T = \frac{q_1}{p_1} = \frac{1}{\Phi'}$ and it can be understood as the pesent value of expected marginal profit of the firm that uses one additional unity of capital. When the actualised market value of the firm correponds to it's true value (no speculation) then Tobin's marginal

[¶]We suppose that the utility function is linear in consumption: Garnier, Venditti and Nishimura[11] have shown that the parameter preference has to be small (close to 0) to allow indeterminacy in the two sector model with sector-specific externalities.

Q equals one. In this model, there is only Tobin's marginal Q of the firm of the consumption sector that can be different from one as only it faces adjustement costs: q_T is the Tobin's marginal Q of the consumption sector firm. If $q_T > 1$ the firm is incited to invest more in capital, in this case the market gives some value to non measurable things. Indeed, because of the non efficiency of the fiancial market, there is some speculation that gives a gap between the true value of the assets of the firme and their expected values. If $q_T < 1$, the explanation is reversed and the firm is incited to diminish its investment.The assumption1 ensures that $q_T = 1$ only if $\frac{y_1}{x_1} = g$, what is true at the steady state.

Moreover, as we have $y_1 = y_1(x_1, p_1)$ then $q_T = q_T(x_1, p_1)$. we differenciate q_T what gives: $\dot{q}_T = \frac{\partial q_T}{\partial x_1}\dot{x}_1 + \frac{\partial q_T}{\partial p_1}\dot{p}_1$ and using $q_T(x_1, p_1) = \frac{1}{\Phi'\left(\frac{y_1(x_1,p_1)}{x_1}\right)}$ we obtain:

$$\frac{\partial q_T}{\partial x_1} = -\frac{1}{x_1}\left(\frac{\partial y_1}{\partial x_1} - \frac{y_1}{x_1^2}\right)\frac{\Phi"}{\Phi'^2}$$
$$\frac{\partial q_T}{\partial p_1} = -\frac{1}{x_1}\frac{\partial y_1}{\partial p_1}\frac{\Phi"}{\Phi'^2}$$

We introduce the following elasticities:

$$\varphi = -\frac{\frac{y_1}{x_1}\Phi''}{\Phi'} : \text{the elasticity of the investment adjustment costs}$$
$$\epsilon_{yx} = \frac{\partial y_1}{\partial x_1}\frac{x_1}{y_1} : \text{the elasticity of the investment good to capital}$$
$$\epsilon_{yp} = \frac{\partial y_1}{\partial p_1}\frac{p_1}{y_1} : \text{the price elasticity of the investment good}$$
$$\epsilon_{wp} = \frac{\partial w_1}{\partial p_1}\frac{p_1}{w_1} : \text{the price elasticity of the interest rate}$$
$$\epsilon_{qx} = \frac{\partial q_T}{\partial x_1}\frac{x_1}{q_T} : \text{the elasticity of the Tobin's marginal Q to capital}$$
$$\epsilon_{qp} = \frac{\partial q_T}{\partial p_1}\frac{p_1}{q_T} : \text{the price elasticity of the Tobin's marginal Q}$$

On assumption 1 we note that $\varphi \geq 0$. This elasticity can be used to express the degree of the investment adjustment costs. Therefore, φ could be understood as a mesure of the efficiency index of the investment per capita.

Moreover, we note that ϵ_{yx} represents the quantity elasticity which is relied on the Rybczynsky effect $\frac{\partial y_1}{\partial x_1}$ (i.e. quantity effect) and ϵ_{wp} the price elasticity which is relied on he Stolper Samuelson effect $\frac{\partial w_1}{\partial p_1}$ (i.e. price effect), with

$$\epsilon_{yx} = \frac{x_1 a_{00}}{y_1 \left(a_{00}a_{11} - a_{01}a_{10}\right)} \text{ and } \epsilon_{wp} = \frac{p_1 \hat{a}_{00}}{w_1 \left(\hat{a}_{00}\hat{a}_{11} - \hat{a}_{01}\hat{a}_{10}\right)}$$

We can caracterize both elasticities in terms of capital intensity differences across private and social levels as in Benhabib and Nishimura [7]. Using the input coefficients given 6 and7, we give the following definition:

Definition 1 *: The consumption good is said to be:*

i) *capital (labor) intensive at the private level if and only if:*

$$a_{00}a_{11} - a_{01}a_{10} < (>)0$$

ii) *capital (labor) intensive at the social level if and only if:*

$$\hat{a}_{00}\hat{a}_{11} - \hat{a}_{10}\hat{a}_{01} < (>)0$$

3 The competitive equilibrium

To obtain the dynamic equations characterizing the equilibrium path, we combine (10) and (11) (after a total differentiation of (10)) and we obtain two equations of motion which describe the dynamic of equilibrium paths[‖]

$$\begin{aligned} \frac{\dot{x}_1}{x_1} &= x_1 \Phi & (14) \\ \frac{\dot{p}_1}{p_1} &= \frac{1}{E}\left[\rho + \Phi'\left(\frac{y_1}{x_1} - \frac{w_1}{p_1}\right) - \Phi + \epsilon_{qx}\frac{\dot{x}_1}{x_1}\right] & (15) \end{aligned}$$

with $E = 1 + \epsilon_{qp}$.

Any solution $\{x_1(t), p_1(t)\}_{t \geqslant 0}$ of the system (14) satisfying the transversality condition (13) will be called equilibrium path.

[‖]We note that all function depends on x_1 and p_1 :$w_1 = w_1(p_1)$, $y_1 = y_1(x_1, p_1)$ and $\Phi = \Phi\left(\frac{y_1(x_1,p_1)}{x_1}\right)$

3.1 Steady state

We want to study the dynamical system (14) in the neighborhood of the steady state.

Proposition 1 *Under assumption 1 there exists a unique steady state* $(x_1^*, p_1^*) > 0$ *solution of :*

$$\begin{aligned}
\dot{x}_1 &= 0 \Longleftrightarrow \frac{y_1(x_1, p_1)}{x_1} = g \\
\dot{p}_1 &= 0 \Longleftrightarrow \frac{w_1(p_1)}{p_1} = \rho + g
\end{aligned}$$

On the assumption 1 the steady state is the same that the one of the model with linear capital accumulation and $\varphi = -g\Phi''(g)$ and the Tobin's marginal Q evaluated at the steady state gives $q_T^* = 1$.

Moreover, it's possible to express conditions i) and ii) of the definition 1 only with the technological parameters β_{ij} and $\hat{\beta}_{ij}$.

Proposition 2 *: Let* $b \equiv \beta_{00}\beta_{11} - \beta_{01}\beta_{10}$ *and* $\hat{b} \equiv \hat{\beta}_{00}\hat{\beta}_{11} - \hat{\beta}_{10}\hat{\beta}_{01}$. *At the steady state we have:*

i) $a_{00}a_{11} - a_{01}a_{10} < (>)0 \Leftrightarrow b < (>)0$

ii) $\hat{a}_{00}\hat{a}_{11} - \hat{a}_{10}\hat{a}_{01} < (>)0 \Leftrightarrow \hat{b} < (>)0$

It follows that $\partial y_1/\partial x_1$ corresponds to the factor intensity difference from the private viewpoint (Rybczynski effects), while $\partial w_1/\partial p_1$ corresponds to the factor intensity difference from the social viewpoint (Stolper-Samuelson effects). At the steady state we have:

$$\begin{array}{c}
\epsilon_{yx} = \frac{\rho+g}{gb} \\
\epsilon_{wp} = \frac{1}{\hat{b}} \\
\epsilon_{yp} = -\frac{\rho+g}{g}\frac{\beta_{00}}{b}\left(\frac{2\hat{\beta}_{10}-1}{\hat{b}}\right) - 1 + \frac{\hat{\beta}_{10}}{\hat{b}}
\end{array} \qquad (16)$$

Moreover, we have the following relationship between elasticities:

$$
\begin{aligned}
\epsilon_{qx} &= \varphi\left(\epsilon_{yx}-1\right)\\
\epsilon_{qp} &= \varphi\epsilon_{yp}
\end{aligned}
\tag{17}
$$

When the firm of the investment sector faces to adjustment costs there is a link between the elasticity of the Tobin's marginal Q to capital ϵ_{qx} and the quantity effect called Rybczynsky effect, measured by ϵ_{yx} and between the price elasticity of the Tobin's marginal Q ϵ_{qp} and the price effect called Stolper Samuleson effect, measured by ϵ_{wp} through the relationship between the price elasticity of the investment good ϵ_{yp} and ϵ_{wp}. Finally, all the elasticities are link to the both Rybczynsky effect and Stolper Samuelson effect and the adjustment costs create a link between these effects and the variation of the Tobin's marginal Q.

The existence of these costs disturbes the market through the impact of the investment on the capital stock and on the output that leads to difference between the true value of the asset of the firm and its evaluation of the financial market. As we will see, the expectation of this gap by the agents is the source of the endogenous fluctuations in this model.

3.2 The linearized system

In order to study the indeterminacy properties of equilibrium, we linearize the system (14) around (x_1^*, p_1^*) which gives the following Jacobian matrix:

$$
J = \begin{pmatrix}
\frac{g\epsilon_{qx}}{\varphi} & g\left(\frac{x_1}{p_1}\frac{\epsilon_{qp}}{\varphi}-1\right)\\
\frac{\frac{p_1}{y_1}g(g+\rho)\epsilon_{qx}(1-\epsilon_{wp})}{E*} & \frac{\left\{(\delta+g)(1-\epsilon_{wp})-\frac{p_1}{y_1}\left[g^2\epsilon_{qx}\left(\frac{x_1}{p_1}\epsilon_{qp}-1\right)\right]+\rho\epsilon_{qp}\right\}}{E*}
\end{pmatrix}
\tag{18}
$$

where all the elasticities are evaluated at the steady state.

Given initial capital stock $x_1(0)$ if there is more than one initial price $p_1(0)$ in the stable manifold of (x_1^*, p_1^*), the equilibrium path coming from $x_1(0)$ will not be unique. In particular, if the Jacobian matrix J (18) has two eigenvalues with negative real part (the locally stable manifold of the steady state (x_1^*, p_1^*) is two dimensional), there will be a continuum of converging paths and thus a continuum of equilibria: (x_1^*, p_1^*) is said to be locally indeterminate.

The dynamic of the model around the steady state can be fully derived from the eigenvalues of Jacobian matrix (18). If we denote T and D the trace and the determinant of the Jacobian matrix (18), we know that the steady state is locally indeterminate if and only if $T < 0$ et $D > 0$. Therefore, we need to study the sign of T and D given by:

$$T = \frac{1}{E}\left\{g\epsilon_{qx}\left(\frac{1}{\varphi}+\frac{p_1}{x_1}\right)+(\rho+g)(1-\epsilon_{wp})+\rho\epsilon_{qp}\right\} \tag{19}$$

$$D = \frac{1}{E}g\epsilon_{qx}\left(\frac{\rho+g}{\varphi}(1-\epsilon_{wp})+\rho\frac{p_1}{x_1}\right) \tag{20}$$

with $E = 1+\epsilon_{qp}$.

4 Existence of local indeterminacy

Our main objective is to study the impact of adjustment costs measured by the elasticity φ on the indeterminacy mechanism coming from sector specific externalities.

Solving the system (19-20) with respect to φ gives a linear relationship between $T(\varphi)$ and $D(\varphi)$: when φ varies on $[0,\infty[$, $T(\varphi)$ and $D(\varphi)$ move along the line called in what follows Δ_φ, which is defned by**:

$$D = S_\varphi T + M_\varphi$$

with

$$S_\varphi = \frac{\frac{\partial D}{\partial\varphi}}{\frac{\partial T}{\partial\varphi}} = \frac{g(\rho+g)(\epsilon_{yx}-1)\left[\frac{p_1}{x_1}\frac{\rho}{\rho+g}-\epsilon_{yp}(1-\epsilon_{wp})\right]}{g\frac{p_1}{x_1}(\epsilon_{yx}-1)+\rho\epsilon_{yp}+\epsilon_{yp}\left(g(\epsilon_{yx}-1)+(\rho+g)(1-\epsilon_{wp})\right)} \tag{21}$$

$$M_\varphi = \frac{\left[g\left(\frac{\partial y_1}{\partial x_1}-g\right)+\rho\frac{\partial y_1}{\partial p_1}\right]\left(\frac{\partial y_1}{\partial x_1}-g\right)\left(\rho+g-\frac{\partial w_1}{\partial p_1}\right)-\rho g\left(\frac{\partial y_1}{\partial x_1}-g\right)\left(\rho+g-\frac{\partial w_1}{\partial p_1}\right)}{g\frac{p_1}{x_1}(\epsilon_{yx}-1)+\rho\epsilon_{yp}+\epsilon_{yp}\left(g(\epsilon_{yx}-1)+(\rho+g)(1-\epsilon_{wp})\right)} \tag{22}$$

Note that S_φ and M_φ depend only upon technological parameters, discount rate and depreciation rate.

We use the geometrical method of Grandmont, Pintus and De Vilder [12] in order to study the variations of $T(\varphi)$ and $D(\varphi)$ in the (T,D) plane, when φ varies continuously on $[0,\infty[$.

**Note that (x_1^*, p_1^*) does not depend on φ and remains the same along line Δ_α.

4.1 Without adjustment costs i.e. $\varphi = 0$

In the case of $\varphi = 0$ there is no adjustment costs what it's correspond to the linear accumulation of capital, we get $E = 1$, $\epsilon_{qx} = \epsilon_{qp} = 0$ and:

$$T(0) = g\left(\epsilon_{yx} - 1\right) + \left(\rho + g\right)\left(1 - \epsilon_{wp}\right) \text{ and } D(0) = g\left(\rho + g\right)\left(\epsilon_{yx} - 1\right)\left(1 - \epsilon_{wp}\right)$$

Therefore, we can give the indeterminacy condition given by Benhabib and Nishimura [7] in the two-sectors model with exogenous labor, linear utility function, sector specific externalities and linear capital accumulation: the steady state is locally indeterminate if and only if the consumption good is capital intensive at the private level ($b < 0$) and labor intensive at the social level ($\hat{b} > 0$) i.e. there is a factor intensities reversal between the private and the social perspective. This factor intensities reverseal corresponds to a break of the duality between Rybczynszy and Stolper Samuelson effects.

When ($b < 0$) and ($\hat{b} > 0$) the pair $(T(\varphi), D(\varphi))$ begins to move along the Δ_φ half-line in the indeterminacy area but it is possible that the pair $(T(\varphi), D(\varphi))$ may be not in the indeterminacy area when $\varphi = 0$ and get in for a positive value of φ. Therefore we have to study the parameters of the Δ_φ half-line and their end points set (i.e. when $\varphi \to +\infty$. We can do the following assumption to restrict the number of case to study:

Assumption 2 *:* $b < 0$

This assumption can by justify by the fact that the labor share, the capital share and the level of decreasing returns are such that we have a consumption good likely more intensive in capital at the private level than the investment good i.e $b < 0$.

Consequently, on the assumption 2 we have two case to study: $\hat{b} > 0$ and $\hat{b} < 0$. The first case i.e. $\hat{b} > 0$ corresponds to the conditions given by Benhabib and Nishimura [7] to have indeterminancy with standard capital accumulation that is when $\varphi = 0$ i.e. $T(0) < 0$ and $D(0) > 0$. In the second case i.e. $\hat{b} < 0$ we can have have $T(0) < 0$ or $T(0) > 0$ and $D(0) < 0$ and the indeterminacy is ruled out for $\varphi = 0$.

4.2 Infinite degree of adjustment costs i.e. $\varphi \to +\infty$

In the case of infinite degree of adjustment costs i.e. $\varphi \to +\infty$ the Trace and the Determinant (19,20) become:

$$T(\infty) = g\tfrac{x_1}{p_1}\tfrac{\epsilon_{qx}}{\epsilon_{qp}} + \rho$$
$$D(\infty) = g\rho\tfrac{x_1}{p_1}\tfrac{\epsilon_{qx}}{\epsilon_{qp}}$$

We can see immediatly that this line $(T(\infty), D(\infty))$ can not be in the indeterminacy area of the (T, D)plane for any values of b and $\hat{b}$. On the assumption2 and considering equations (16) and (17), we have $\epsilon_{qx} < 0$ when $b < 0$. Finally, the both sign of $T(\infty)$ and $D(\infty)$ are only relied on the sign of the elasticityϵ_{qp} and more precisely on ϵ_{yp}. The following proposition gives sufficient conditions to have $\epsilon_{yp} > 0$ or $\epsilon_{yp} < 0$ i.e. $\epsilon_{qp} > 0$ or $\epsilon_{qp} < 0$.

Proposition 3 *: On the assumption2, when $\hat{\beta}_{10} > 1/2$, ϵ_{qp} and $\hat{b}$ have the same sign.*

When $\hat{b} < 0$ and $b < 0$ (2) this proposition collapses to a level of sector specific externalities such that we have no capital intensity reversal by opposition to the case $\hat{b} > 0$. This proposition will allow us to know the location of $(T(\infty), D(\infty))$ in the (T, D) plane, for any value of b and $\hat{b}$.

To ensure that the proposition 3 is checked we assume that the capital share in the social production function of consumption sector is greater than 50%:

Assumption 3 *: $\hat{\beta}_{10} > 1/2$.*

Now we have all that we need to study $(T(\varphi), D(\varphi))$ for $\varphi > 0$.

4.3 General case: finite degree of adjustment costs i.e. $\varphi > 0$

Finally, on assumptions 2 , when $\varphi = 0$, $(T(\varphi), D(\varphi))$ is in the indeterminacy area and when $\varphi \to +\infty$ it is in positive or negative area of the (T, D) plane.

Consequently, when $\varphi*$ increases from 0 to $+\infty$, the pair $(T(\varphi), D(\varphi))$ moves along the line $\triangle_\varphi$ from the starting point such that $T(0)$and $D(0)$ to the end point $(T(\infty), D(\infty))$. We have two possibilities, let $T(0) < 0$ and

$D(0) > 0$ i.e. the starting point is in the indeterminacy area and let $T(0) < 0$ or $T(0) > 0$ and $D(0) < 0$ i.e. the starting point is out of the indeterminacy area.

In the first case, that corresponds to the case where the indeterminancy needs a capital intensity reversal between the private and social level that is $b < 0$ and $\hat{b} > 0$ (Benhabib and Nishimura [7]), it's easy to give the conditions for indeterminacy when $\varphi > 0$:

Proposition 4 *: On the assumption 1,2 and 3, when $\hat{b} > 0$ the proposition 3 ii) is verified and $\exists \overline{\varphi} > 0$ such that:*

i) $\forall \ \varphi \in \left]0, \overline{\varphi}\right[\ : \ T(\varphi) < 0$ *and* $D(\varphi) > 0$ *and the steady state is locally indeterminate*

ii) $T(\overline{\varphi}) = 0$ *and* $D(\overline{\varphi}) > 0$ *and there is a Höpf bifurcation.*

The case i) is in the way of the indeterminacy condition given by Benhabib and Nishimura [7] and extends it to the presence of adjustment cost. The indeterminacy mechanism needs less small adjustment costs. Indeed, if the adjustment costs were so much high they will kill this mechanism by absoption effect of the impact of the rate of return of capital on the price of the investment good. In the case ii) the level of adjustment costs offset exactely the impact of the externalities on the rate of return of capital and the price of the investment good go back exactly to his first value: a cycle can appear.

When there is no capital intensity reversal, that is $b < 0$ and $\hat{b} < 0$, it's more difficult since the starting point is out of the indeterminacy area and we have to ensure that $(T(\varphi), D(\varphi))$ gets inside. One of the possibilities is to analyse the parameter S_φ of the equation of the half line $\triangle_\varphi$ which is equivalent to study the derivatives $\frac{\partial T}{\partial \varphi}$ and $\frac{\partial D}{\partial \varphi}$. If $\frac{\partial D}{\partial \varphi} > 0$ and $\frac{\partial T}{\partial \varphi} < 0$, when φ increases from 0 to $+\infty$, the pair $(T(\varphi), D(\varphi))$ starts from $T(0) > 0$ and $D(0) < 0$ (the case where $T(0) < 0$ is ruled out: see proof 7.5 of proposition 5) and moves along the half-line $\triangle_\varphi$ in the direction of the indeterminacy area, gets in for a lower bound $\underline{\varphi}$ and gets out for a upper bound $\overline{\varphi_2}$. We can give the second proposition:

Proposition 5 *:On the assumption 1,2 and 3, when* $0 > \hat{b} > b$ *the proposition 3 i) is verified and* $\exists(\underline{\varphi}, \overline{\varphi}_2)$ *with* $\overline{\varphi}_2 > \underline{\varphi} > 0$ *such that:*

i) $\forall\ \varphi \in \left]\underline{\varphi}, \overline{\varphi}_2\right[\ :\ T(\varphi) < 0$ *and* $D(\varphi) > 0$ *and the steady state is locally indeterminate*

ii) $T(\overline{\varphi}_2) = 0$ *and* $D(\overline{\varphi}_2) > 0$ *and there is a Höpf bifurcation.*

and $\underline{\varphi} = -1/\epsilon_{yp}$

This proposition shows that the local indeterminacy of the steady state is possible without factor intensities reversal between the private and social level. Indeed, when the adjustment cost is sufficiently high and the consumption good is capital intensive at the private and social levels, there is a room for local indeterminacy of the steady state.

4.3.1 Economic intuition

Now, we try to explain the economic intuition of the impact of the adjustment costs on the indeterminacy mechanism without factor intensities reversal between the private and social level.

Starting from an arbitrary equilibrium, suppose that the agents change collectively their expectations and believe that the price of investment good will increase, that is $q_T < 1$. If fluctuations exist we may find another converging trajectory to the steady state which implies the dynamic of q_T or the price p_1 is not explosive.

As consequence of this belief, the investment rate decreases i.e. y_1 decreases and its price p_1 increases (that confirmes the belief of agents). A lower investment rate results in lower capital stock x_1 (because of adjustment costs this effect is smaller than the case without adjustment costs). When the consumption good is capital intensive at the private level a decrease in capital stock decreases its output at constant price by the Rybczynsky effect and through a transfer of labor from the consumption sector to the investment sector (as the labor is exogenous and normalized to one), the output of the investment sector increases. But this increase is low since the small decrease of capital stock because of adjustment costs. The ratio $\frac{y_1}{x_1}$ increases and as

$\Phi'' < 0$ and the marginal Q of Tobin $q_T = 1/\Phi'$ too. The expected value of the assets of the firm in the investment sector is greater than its true value: the financial markets over-evaluate the asset of this firm. Now, we need a mechanism which reverses the price p_1 towards the equilibrium (to avoid a price explosion) and offsets this initial expected increase. From the Stolper-Samuelson theorem, the initial rise of the price p_1 leads to a decrease of marginal rate of return of capital w_1. The question is: how can we get a reverse of price p_1? We have to consider the relationship between the overall of return given by the following equation:

$$\frac{\dot{p}_1}{p_1}(1 + \epsilon_{qp}) + \frac{w_1}{p_1} = \rho + g \tag{23}$$

Since $\frac{w_1}{p_1}$ decreases, to maintain the equality of the overall return to capital to $\rho + g$, the term $\frac{\dot{p}_1}{p_1}(1 + \epsilon_{qp})$ has to increase sufficiently but the proposition 5 implies that $1 + \epsilon_{qp} < 0$ and we have to check that $\frac{\dot{p}_1}{p_1} < 0$. Consequently the price p_1 must to decline and with the effect of adjustment cost, it is enough to check the Rybczynsky effect, the initial rise of p_1 is offseted. This decrease of the price leads to an increase of the expected value of the asset of the firm and the marginal Q of Tobin q_T increases, that leads to further increase of the investment level. Moreover, through the long term link between the marginal Q of Tobin and the financial market and because of the expectation of agents, the movments of the marginal Q of Tobin will be followed by the financial markets.

Finally, adjustment costs reduce the response of capital stock to a movement of investment and reduce the Rybczynsky effect and allow Stolper Samuelson effect to play in the good way to offset the Rybczynsky effect.

Therefore, when there are sufficiently high adjustment costs, the duality between Rybczynsky and Stolper-Samuelson effects is not broken. Moreover, we can see that this mechanism of endogenous fluctuations is linked to a fluctuation of the financial market through the expected value of the asset of the firm that is the value given by the market to the asset of the firm. Therefore the presence of adjustment costs leads to a gap between the true value of the asset of the firm and its expected value given by the financial market, this gap is the key of this new mechanism of endogenous fluctuations.

5 Calibration examples

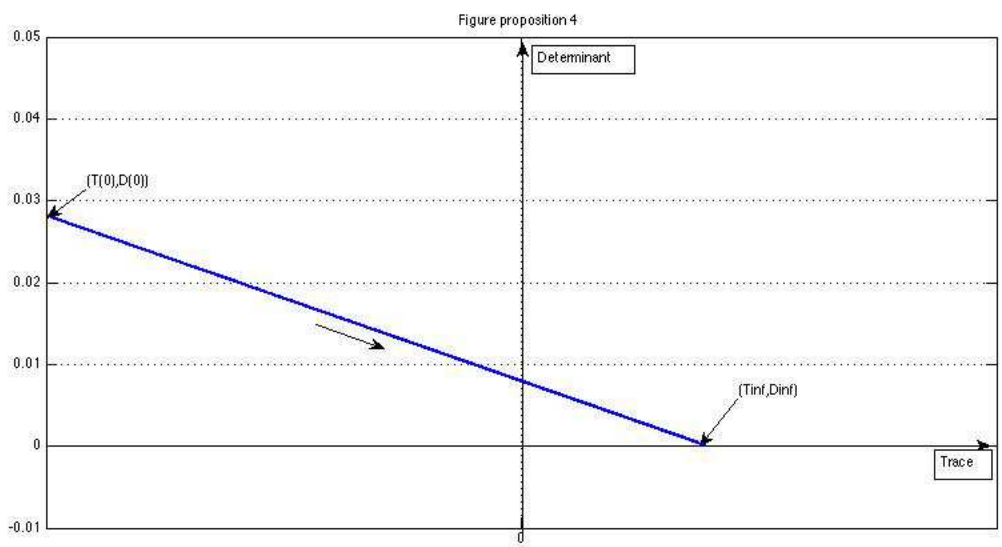
Benhabib and Nishimura [7] show that the level if sector specific externalities can be small (almost 5%) with only a labor externality in the consumption sector. Let's try to illustrate ours propositions with the lower levels of esternalities as possible. Following Benhabib and Nishimura we take only a labor externality in the consumption sector.

5.1 Example for proposition 4

We consider the following parametrization:

$\hat{\beta}_{00} = 0.37$	b_{00} =0.05	$\hat{\beta}_{10} = 0.63$	b_{10} =0
$\hat{\beta}_{01} = 0.34$	$b_{01} = 0$	$\hat{\beta}_{11} = 0.66$	$b_{11} = 0$

We find that $\overline{\varphi} = 0.155$

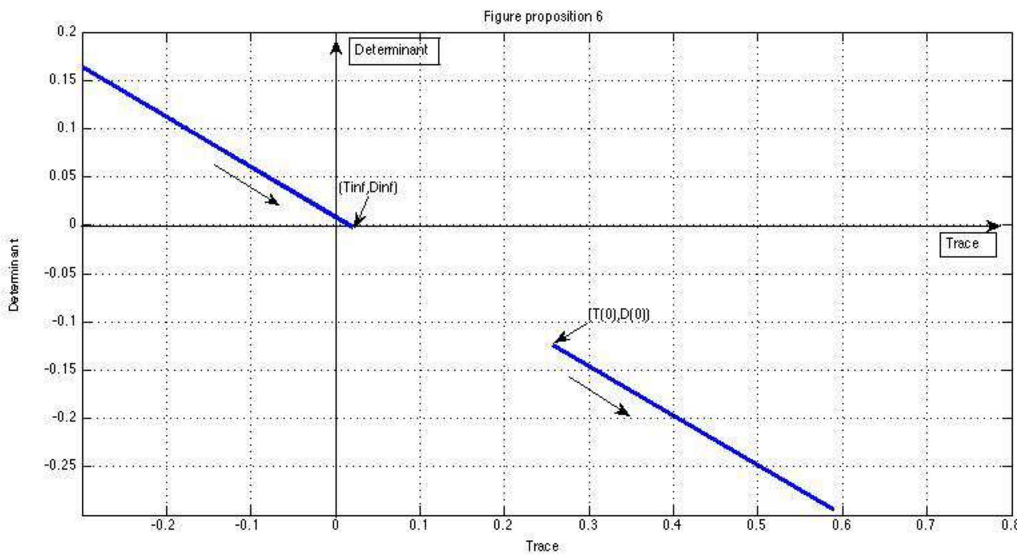


5.2 Example for proposition 5

We consider the following parametrization:

$\hat{\beta}_{00} = 0.34$	$b_{00} =0.05$	$\hat{\beta}_{10} = 0.66$	$b_{10} =0$
$\hat{\beta}_{01} = 0.4$	$b_{01} = 0$	$\hat{\beta}_{11} = 0.60$	$b_{11} = 0$

We find that $\overline{\varphi} =$



6 Concluding comments

In this paper we have prove that concave adjustment costs function interplays with sector specific externalities to lead to endogenous fluctuations.

7 Appendix

7.1 Proof of existence of (x_1^*, p_1^*)

The maximization of profit gives the following first order conditions :

$$w_i = p_j \beta_{ij} \frac{y_j}{x_{ij}} \quad \text{for} \quad i, j = 0, 1 \tag{24}$$

The steady state is characterized by: $y_1 = g x_1$ and $w_1 = (\rho + g) p_1$, so that:

$$x_{11} = \frac{\beta_{11}}{\rho + g} g x_1 \tag{25}$$

Moreover we have:

$$x_{01} = gx_1 \left(\frac{\beta_{11}}{\rho + g} \right)^{-\frac{\hat{\beta}_{11}}{\hat{\beta}_{01}}} \tag{26}$$

The stock equations: $x_1 = x_{10} + x_{11}$ et $1 = x_{01} + x_{00}$ allow to give:

$$x_{10} = x_1 \left(1 - g \frac{\beta_{11}}{\rho + g} \right) \tag{27}$$

$$x_{00} = 1 - gx_1 \left(\frac{\beta_{11}}{\rho + g} \right)^{-\frac{\hat{\beta}_{11}}{\hat{\beta}_{01}}} \tag{28}$$

From (24) we have:

$$\frac{x_{00}x_{11}}{x_{01}x_{10}} = \frac{\beta_{00}\beta_{11}}{\beta_{01}\beta_{10}} \tag{29}$$

From (25), (26), (27), (28), (29), we have:

$$x_1^* = \frac{\left(\frac{\beta_{11}}{\rho+g} \right)^{-\frac{1}{\hat{\beta}_{01}}}}{\frac{\beta_{00}\beta_{11}}{\beta_{01}\beta_{10}} + g \left(1 - \frac{\beta_{00}\beta_{11}}{\beta_{01}\beta_{10}} \right) \frac{\beta_{11}}{\rho+g}} > 0 \tag{30}$$

For $i = 1$and $j = 0$, (24), (27) and (30) give:

$$w_1^* = (\rho + g)\, p_1^* \tag{31}$$

We derive:

$$p_1^* = \beta_{10} \left(\frac{\beta_{00}\beta_{11}}{\beta_{01}\beta_{10}} \right)^{\hat{\beta}_{00}} \left(\frac{\hat{\beta}_{11}}{\delta + g} \right)^{-\frac{\hat{\beta}_{00}}{\hat{\beta}_{01}}} > 0 \tag{32}$$

7.2 Computation of derivatives used in $T(\alpha)$ and $D(\alpha)$

In order to compute (19) and (20) we need the following partial derivatives: $\frac{\partial y_1}{\partial x_1}$, $\frac{\partial c}{\partial x_1}$, $\frac{\partial w_1}{\partial p_1}$,$\frac{\partial y_1}{\partial p_1}$, $\frac{\partial c}{\partial p_1}$.

To compute $\frac{\partial y_1}{\partial p_1}$and $\frac{\partial c}{\partial p_1}$ we begin by the total differentiation of the quantity equations given by:

$$\begin{aligned} a_{00}y_0 + a_{01}y_1 &= 1 \\ a_{10}y_0 + a_{11}y_1 &= x_1 \end{aligned}$$

The total differentiation gives:

$$a_{00}dy_0 + a_{01}dy_1 + \frac{\partial a_{00}}{\partial w_0}y_0 dw_0 + y_1\left(\frac{\partial a_{01}}{\partial w_0}dw_0 + \frac{\partial a_{01}}{\partial p_1}dp_1\right) = 0 \quad (33)$$

$$a_{10}dy_0 + a_{11}dy_1 + \frac{\partial a_{10}}{\partial w_1}y_0 dw_1 + y_1\left(\frac{\partial a_{11}}{\partial w_1}dw_1 + \frac{\partial a_{11}}{\partial p_1}dp_1\right) = dx_1 \quad (34)$$

After, we need $\frac{\partial c}{\partial x_1}$ and $\frac{\partial y_1}{\partial x_1}$ with $c = y_0$ and $dw_0 = dw_1 = dp_1 = 0$. Then, we have:

$$\frac{\partial y_1}{\partial x_1} = \frac{a_{00}}{a_{11}a_{00} - a_{10}a_{01}} \quad (35)$$

$$\frac{\partial c}{\partial x_1} = -\frac{a_{01}}{a_{11}a_{00} - a_{10}a_{01}} \quad (36)$$

These derivatives correspond to the Rybczynsky effect.

Now, compute $\frac{\partial y_1}{\partial p_1}$ and $\frac{\partial c}{\partial p_1}$. With the price equations given by:

$$\begin{aligned} \hat{a}_{00}w_0 + \hat{a}_{10}w_1 &= 1 \\ \hat{a}_{01}w_0 + \hat{a}_{11}w_1 &= p_1 \end{aligned}$$

So:

$$\frac{\partial w_1}{\partial p_1} = \frac{\hat{a}_{00}}{\hat{a}_{11}\hat{a}_{00} - \hat{a}_{10}\hat{a}_{01}} \quad (37)$$

$$\frac{\partial w_0}{\partial p_1} = -\frac{\hat{a}_{10}}{\hat{a}_{11}\hat{a}_{00} - \hat{a}_{10}\hat{a}_{01}} \quad (38)$$

And using proposition 2 we have:

$$\frac{\partial y_1}{\partial x_1} = \frac{\beta_{00}\left(\rho + g\right)}{b}; \frac{\partial c}{\partial x_1} = -\frac{\beta_{01}w_1}{b}; \frac{\partial w_1}{\partial p_1} = \frac{\hat{\beta}_{00}\left(\rho + g\right)}{\hat{b}}; \frac{\partial w_0}{\partial p_1} = \frac{\hat{\beta}_{10}w_0}{p_1\hat{b}} \quad (39)$$

On the other hand we derive from

$$a_{ij}(w_1, p_j) = \frac{\beta_{ij} p_j}{w_i} = \frac{x_{ij}}{y_j}$$

that:

$$\begin{aligned} \frac{\partial a_{ij}}{\partial w_i} &= -\frac{a_{ij}}{w_i} && (40) \\ \frac{\partial a_{ij}}{\partial p_j} &= \frac{a_{ij}}{p_j} && (41) \end{aligned}$$

We substitute (37), (38), (39) et (41) in (33) and (34). Hence, the resolution of the system (33) and (34), with $dx_1 = 0$ give:

$$\begin{aligned} a_{00} dy_0 + a_{01} dy_1 + dp_1 \left(\frac{a_{01}}{p_1} y_1 - \left(p_1 - \frac{\hat{a}_{11}}{\hat{a}_{10}} \right)^{-1} \right) &= 0 && (42) \\ a_{10} dy_0 + a_{11} dy_1 + dp_1 \left(\frac{a_{11}}{p_1} y_1 - x_1 \left(p_1 - \frac{\hat{a}_{01}}{\hat{a}_{00}} \right)^{-1} \right) &= 0 && (43) \end{aligned}$$

Using (42) and (43) we have thus:

$$\begin{aligned} \frac{\partial y_1}{\partial p_1} &= -\frac{x_1}{p_1} \left\{ \frac{\beta_{00} (\rho + g)}{b} \left(\frac{2\hat{\beta}_{10} - 1}{\hat{b}} \right) + g \left(1 - \frac{\hat{\beta}_{10}}{\hat{b}} \right) \right\} && (44) \\ \frac{\partial c}{\partial p_1} &= x_1 (\rho + g) \left\{ \frac{\beta_{01}}{b} \left(\frac{2\hat{\beta}_{10} - 1}{\hat{b}} \right) - \frac{1}{\beta_{10}} \left(1 - g \frac{\beta_{11}}{\rho + g} \right) \frac{\hat{\beta}_{10}}{\hat{b}} \right\} && (45) \end{aligned}$$

Using constant returns to scale at the social level and proposition 2 we have:

$$\hat{b} = \hat{\beta}_{00} - \hat{\beta}_{01} = \hat{\beta}_{11} - \hat{\beta}_{10} \qquad (46)$$

At the steady state, using 46, 39 and 42 we can deduct that:

$$\epsilon_{yx} = \frac{\rho+g}{gb}$$
$$\epsilon_{wp} = \frac{1}{\hat{b}}$$
$$\epsilon_{yp} = -\frac{\rho+g}{g} \frac{\beta_{00}}{b} \left(\frac{2\hat{\beta}_{10}-1}{\hat{b}} \right) - 1 + \frac{\hat{\beta}_{10}}{\hat{b}}$$

7.3 Computation of Δ_α

Consider the expressions $T(\varphi)$ (19), $D(\varphi)$ (20) and $E = 1 + \varphi\epsilon_{yp}$

With (19) and (20) we can extract α

$$\varphi = \frac{g(\rho+g)(\epsilon_{yx}-1)(1-\epsilon_{wp}) - D(\varphi)}{\epsilon_{yp}D(\varphi) - g\rho\frac{p_1}{x_1}(\epsilon_{yx}-1)} = \frac{g(\epsilon_{yx}-1) + (\rho+g)(1-\epsilon_{wp}) - T(\varphi)}{\epsilon_{yp}T(\varphi) - g\frac{p_1}{x_1}(\epsilon_{yx}-1) - \rho\epsilon_{yp}} \tag{47}$$

Therefore:

$$D = S_\varphi T + M_\varphi \tag{48}$$

with S_φ and M_φ are given by (21).

The computation of the derivative $\frac{dT}{d\alpha}$ and $\frac{dD}{d\alpha}$ give :

$$\frac{dT}{d\varphi} = \frac{1}{E^2}\left[g\frac{p_1}{x_1}(\epsilon_{yx}-1) + \rho\epsilon_{yp} - \epsilon_{yp}\left(g(\epsilon_{yx}-1) + (\rho+g)(1-\epsilon_{wp})\right)\right] \tag{49}$$

$$\frac{dD}{d\varphi} = \frac{g(\rho+g)(\epsilon_{yx}-1)\left(\frac{p_1}{x_1}\frac{\rho}{\rho+g} - \epsilon_{yx}(1-\epsilon_{wp})\right)}{E^2} \tag{50}$$

When technological parameters are fixed, only E depends on φ in the expression of these derivatives. Hence, the sign of $\frac{\partial T}{\partial \alpha}$ and $\frac{\partial D}{\partial \alpha}$ does not depend on φ and remains constant $\forall\ \varphi$.

7.4 Proof of Proposition 3

On the assumption 2 i.e. $b < 0$ and using $\epsilon_{yp} = -\frac{\rho+g}{g}\frac{\beta_{00}}{b}\left(\frac{2\hat{\beta}_{10}-1}{\hat{b}}\right) - 1 + \frac{\hat{\beta}_{10}}{\hat{b}}$ and equations (16) and (46) when $\hat{\beta}_{10} > 1/2$ we have $\epsilon_{yp} < 0$ if $\hat{b} < 0$ (i.e. $\hat{\beta}_{10} > \hat{\beta}_{11}$) and $\epsilon_{yp} > 0$ if $\hat{b} > 0$ (i.e.$\hat{\beta}_{10} < \hat{\beta}_{11}$). Finally, using equation (17), we can conclude that, on the assumption 2, if $\hat{\beta}_{10} > 1/2$ then ϵ_{qp} and $\hat{b}$ have the same sign.

7.5 Proof of Proposition 4

On the assumption 2 (i.e. $b < 0$) and $\hat{b} > 0$ we have $\epsilon_{yx} - 1 < 0$ and $1 - \epsilon_{wp} < 0$ that is $T(0) < 0$ and $D(0) > 0$ and the starting point is in the indeterminacy area. If we add assumptions1and 3, when $\hat{b} > 0$ we know from proposition 3 that $\epsilon_{qp} > 0$. Consequently, we have $\frac{dD}{d\varphi} < 0$. To conclude, we have to study the end point $(T(\infty), D(\infty))$.

To have a Höpf bifurcation (i.e. the point $(T(\varphi), D(\varphi))$ crosses the ordonate axe when it gets out the indeterminancy area), we have to ensure that the point $(T(\infty), D(\infty))$ is in the half-plan such $T(\varphi) > 0$ and $D(\varphi) < -T(\varphi)$), that is:

i) $D(\infty) < -T(\infty)$

ii) $T(\infty) > 0$

The point i) comes easily since when $b < 0$ and $\hat{b} > 0$, we can conclude that $D(\infty) = g\rho\frac{x_1}{p_1}\frac{\epsilon_{qx}}{\epsilon_{qp}} < 0$ and $D(\infty) = g\rho\frac{x_1}{p_1}\frac{\epsilon_{qx}}{\epsilon_{qp}} < -g\frac{x_1}{p_1}\frac{\epsilon_{qx}}{\epsilon_{qp}} - \rho = -T(\infty)$.

In order to prove the point ii) we have to study the sign of $T(\infty)$, using equations (16), (46) and (17), we have $T(\infty) > 0$ if:

$$\frac{\rho+g}{gb}\left[g - \rho\frac{p_1}{x_1}\frac{\beta_{00}}{\hat{b}}\left(2\hat{\beta}_{10} - 1\right)\right] - g + \rho\frac{p_1}{x_1}\frac{\left(2\hat{\beta}_{10} - \hat{\beta}_{11}\right)}{\hat{b}} > 0 \qquad (51)$$

On the assumption 2,3 and $\hat{b} > 0$, we have $\frac{\rho+g}{gb}\left[g - \rho\frac{p_1}{x_1}\frac{\beta_{00}}{\hat{b}}\left(2\hat{\beta}_{10} - 1\right)\right] > 0$ and $\rho\frac{p_1}{x_1}\frac{\left(2\hat{\beta}_{10} - \hat{\beta}_{11}\right)}{\hat{b}} > 0$ and then $T(\infty) > 0$.

Finally when φ increases from 0 to ∞, the point $(T(\varphi), D(\varphi))$ starts from $(T(0), D(0))$ in the indeterminacy area, crosses the ordonate axe and finishes on $(T(\infty), D(\infty))$ according a decrease of the $D(\varphi)$ as $\frac{dD}{d\varphi} < 0$.

7.6 Proof of proposition 5

On the assumption 2 (i.e. $b < 0$) and $\hat{b} < 0$ we have $\epsilon_{qx} < 0$ and $1 - \epsilon_{wp} > 0$ that is $T(0) < 0$ or $T(0) > 0$ and $D(0) < 0$. If we add assumptions1and 3, when $\hat{b} < 0$ we know that the proposition 3i) is checked and $\epsilon_{yp} < 0$. Consequently, we have $\frac{dD}{d\varphi} < 0$ and $T(\infty) > 0$ and $D(\infty) > 0$

To have indeterminancy and Höpf bifurcation, as $\frac{dD}{d\varphi} < 0$ we have to ensure that:

i) $T(0) > T(\infty)$

ii) $T(0) > 0$

The point i) is equivalent to:

$$g\left(\frac{\rho+g}{gb}-1\right)+(\rho+g)\left(1-\frac{1}{\hat{b}}\right)>\frac{g(\frac{\rho+g}{gb}-1)+\rho\frac{p_1}{x_1}\epsilon_{yp}}{\frac{p_1}{x_1}\epsilon_{yp}}$$

which gives:

$$\frac{p_1}{x_1}\epsilon_{yp}\left(\frac{1}{b}-\frac{1}{\hat{b}}\right)>\frac{1}{b}-\frac{g}{\rho+g}$$

What is true if $\hat{b}>b$ (with all the hypothesis made before).

To prove the point ii) we have to study the sign of $T(0)$: it's positive if $g(\frac{\rho+g}{gb}-1)+(\rho+g)\left(1-\frac{1}{\hat{b}}\right)>0$ that is, if $\hat{b}>b$.

When 2,3 and $0>\hat{b}>b$ are checked we have $T(0)>T(\infty)>0$ and when φ increases from 0 to ∞, the point $(T(\varphi),D(\varphi))$ starts from $(T(0),D(0))$ gets down and jumps in the indeterminacy area, crosses the ordonate axe and finishes on $(T(\infty),D(\infty))$ according a decrease of the $D(\varphi)$ as $\frac{dD}{d\varphi}<0$.

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