Comparison of Approaches for Optimization of Electromagnetic Devices using Finite Element Method

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In the context of optimization using finite element method (FEM), many issues arise, mainly numerical noise due to re-meshing when dealing with geometric parameters furthermore the computational time of the overall optimization can be prohibitive. In this paper, we compare different approaches to deal with these issues. In some approaches, optimization method is coupled directly to FEM simulation while other approaches aim to replace the expensive simulation with cheap meta-models that reduce the overall time of optimization. As treating different approaches, a comparison protocol is proposed and two electromagnetic optimization benchmarks are treated.

Index Terms—**Finite element method, Meta-model, Optimization methods, TEAM Workshop problems.**

I. Introduction

THE increasing constraints on the design of electromagnetic devices require numerical tools that are able to model the electromagnetic fields. Finite Element (FE) method is the most used method to satisfy such a requirement. However, this method may be very expensive in computational time due to nonlinear behavior, 3D geometries and time dependency. Thus, its usage for optimization, *i.e.* iterative process, should be made with caution since only a limited number of evaluations of the simulation tool is possible. It also suffers from numerical noise due to the re-meshing when dealing with geometric parameters. Thus, these issues have been extensively studied in the literature using meta-model approach and stochastic algorithms [1], *e.g.* genetic algorithms (GA), *etc*. On the other hand, optimization using gradient based algorithms is less studied due to the re-meshing error that highly perturb the computation of gradient using the finite difference technique.

Here, we compare different approaches. The first approach directly use the expensive model. Some methods are derivative-free such as GA and DIRECT, others are gradient-based algorithm, they can also use the derivatives computed using adjoint variable method. The second approach considers the FEM simulation as a black-box and construct cheap meta-models to reduce the computational burden while refining in regions with high error or low objective value.

In this paper, we propose to compare both approaches, and we mainly focus on the implementation and the computational cost to get reliable results. Furthermore, we define metrics for each approach to ease this task.

II. Expensive model Approaches

In these approaches, we can distinguish two categories :

A. Gradient-based

In design optimization, the gradient gives the search direction to gradient based algorithms, *e.g.* sequential quadratic programming, active set... When dealing with FEM models, the gradient information is not always available, it is then approximated using finite difference. However, numerical noise due to re-meshing make this approximation highly sensitive and requires tweaking the step size used for the computation [3]. Thus, the use of such algorithm is not beneficial. To be able to take advantage of such algorithms, a gradient of good quality has to be computed.

The adjoint variable method (AVM) enables to efficiently compute the gradient from FEM code. Thus, no numerical noise due re-meshing is observed as opposed to finite difference. We implemented a 2D FEM code and an AVM code based on the discrete approach [4] for the gradient computation. The implementation of the AVM code is cumbersome since it require intrusive manipulation of the FEM code itself, nevertheless, it remains very interesting in terms of the computational cost, as the time for computing the gradient is equivalent to evaluating FEM code once whatever the number of variables. This can be very advantageous when treating large scale problem, *e.g* eight variables and more.

However, gradient-based algorithms suffer from the convergence to local optima that depends on the initial point. To handle this issue, a multi-start strategy with uniformly distributed initial points is considered. By doing so, we increase the probability of attaining the global optimum. Therefore, we define a convergence rate (C.R.) of the method by considering two solutions as the same if the distance between them is less than a small value. In this study, we took 1% of the design range.

B. Derivative-free

Derivative-free optimization is a discipline in mathematical optimization that does not use derivative information in the classical sense to find optimal solutions. Many of gradient-free algorithms are designed as global optimizers and thus are able to find multiple local optima while searching for the global optimum. Various gradient-free methods have been developed. We are going to look at some of the most commonly used algorithms: genetic algorithm (from Matlab) and DIRECT algorithm [2].

Genetic algorithm is a stochastic optimization method. It is based on randomness inherent in its operators that are selection, mutation and crossover. A convergence rate can be computed and it is expected to be close to one.

The DIRECT method [2] is one of the strategies to do deterministic global optimization, it performs a systematic search of the design space using a hyperdimensional adaptive meshing scheme to search all the design space to find the optimum.

III. Meta-model Approach

Meta-models are widely used in the context of optimization of electromagnetic devices [5][6]. They are constructed based on a small sample from the expensive model, *i.e.* FEM simulation. The optimization is performed on the meta-model rather than the expensive model. This meta-model could be improved using infill criteria that enable to sample new promising designs from the full model. The infill criterion has a strong influence on how efficiently and accurately the algorithm locates the optimum. In this work, we developed a new approach to the usage of meta-models in optimization because the conventional approach of fitting one meta-model and enrich it has some drawbacks :

- The infill criteria that focus more on the local search tend to sample points close to each other which decay the conditioning of the correlation matrix and in consequence the construction of the meta-model [7][8].
- The number of samples exponentially increases as the dimensionality of the optimization problem does (curse of dimension). Thus, the size of the correlation matrix increases and consequently the time needed to fit the meta-model. This time can even exceed the evaluation time of the FE model.
- The infill criteria optimization problem are highly multi-modal and their modality is highly correlated to size of the sample of expensive model, when the number of samples increase the number of local optima increases too. Therefore, it becomes highly difficult to solve these optimization problems [9].
- When dealing with constrained optimization, the infill criteria tend to sample inside the feasible region but not on the constraint boundary which affects the solution accuracy [10].

This approach (B2M2EP) is based on the idea of branch and bound algorithms and consists of a systematic enumeration of candidate solutions by spiting the search space into smaller spaces. The algorithm explores each smaller space then estimates the upper and lower bounds for each one of them. A subspace is discarded if it cannot produce a better solution than the best found so far by the algorithm [11].

IV. Comparison protocol

As comparing different approaches, we propose a probabilistic measure for gradient based algorithms and GA. Once the convergence rate is determined, the probability that at least one optimization lead to the best of all solutions is

$$P = 1 - (1 - C.R.)^n$$

where n is the number of optimizations to be considered. the probability is $P = 99.73\%$, then, we compute n

$$n = \frac{log(1-P)}{log(1-C.R.)}$$

Then, the expected total number of evaluations (Expected # FEM evals) is n times the average number of evaluations.

The stopping criterion for the gradient based approach is based on the step size which is set to 10^{-9}.

For GA, the algorithm is stopped if there is no improvement in the objective function during 50 successive generations.

DIRECT is deterministic method, the implemented stopping criteria are based on a maximum budget, Thus, we stopped it manually when there were no improvement in the objective function in 50 successive iterations.

For B2M2EP, the algorithm is stopped based on depth level criterion, which limit the depth of the search in terms of the size of the smallest sub-space with respect to whole design space.

V. Test cases

A. TEAM Workshop Problem 25

In this problem, the objective is to find the size of the inner die mold and the shape of the outer die mold in order to obtain the desired magnetic field in the cavity shown in the Fig. 1. The optimal radius for the inner mold and the elliptical shape for the outer mold can be determined given specified design objectives [12].

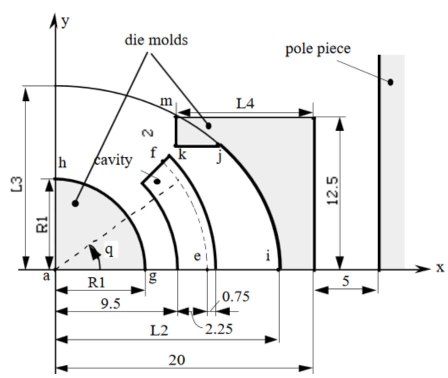

Fig. 1. Enlarged view of die press [12].

The objective of the shape optimization is to obtain a flux density that is radial in the cavity space and with a constant magnitude of $0.35 Tesla$. The objective function W is the squared error between the Bx and By values sampled in 10 positions along the arc e-f.

$$\min_x \quad W(x) = \sum_{i=1}^{10}(B_{xip} - B_{xio})^2 + (B_{yip} - B_{yio})^2 \quad (1)$$

where x are the design variable $x = (R1, L2, L3, L4)$.

The algorithms parameters are as follows

- **Gradient based** : SQP algorithm from Matlab is used with 100 uniformly distributed initial points.
- **GA**: 50 runs are performed, the population size is set 100, selection method is stochastic uniform, crossover method and fraction are scattered and 80% respectively, elite fraction is 5% and mutation method is Gaussian.
- **DIRECT**: Jones factor were set to 10^{-4} while increasing the number of iterations [2].
- **B2M2EP**: we choose to build each meta-model using 8 initial samples generated by composite design and enrich each one of them by 50 samples added using the error prediction [11].

The results are summarized in TABLE I. GA could not get reliable results, only one optimization converged to the solution shown in the table. Furthermore, this one is not competitive compared to other solutions. B2M2EP were able to locate the optimal solution but with higher cost, which is expected since it is based on branch and bound algorithms that enable to get reliable results. Gradient based and DIRECT algorithms gave the best solution, while the former outperforms all the others in term of computational cost.

TABLE I
TEAM WORKSHOP PROBLEM 25 OPTIMIZATIONS RESULTS

Approach	SQP	GA	DIRECT	B2M2EP
R1	7.31	7.51	7.31	7.26
L2	14.21	14.64	14.20	14.07
L3	14.11	14.39	14.08	14.00
L4	14.37	14.44	14.39	14.31
W	7.62e-5	12.44e-5	7.61e-5	9.91e-5
# FEM evals	280	10100	24255	44536
C.R.	34 %	2 %	100 %	100 %
Expected # FEM evals	3585	2146501	24255	44536

It is worth noting that some results from literature [1] were taken into account and compared to ours. But due to the difference in FE model, the optimal values are different. Nevertheless, those from literature, when evaluated in our FE model, have bigger values of the objective function than those presented in TABLE I.

B. TEAM Workshop Problem 22

The SMES device in Fig. 2 consists of two concentric superconducting coils fed with currents that flow in opposite directions [13]. The inner coil is used for storing magnetic energy E, while the outer one has the role of diminishing the magnetic stray field B_{stray}. The goal of the optimization problem is to find the design configurations (8 parameters) that give a specified value of stored magnetic energy and a minimal magnetic stray field while satisfying some constraints. Mathematically, this is formulated as

$$\begin{aligned} \min_x \quad & OF(x) = B^2_{stray}(x)/B^2_{norm} + |E(x) - E_{ref}|/E_{ref} && (2) \\ s.t. \quad & |\mathbf{J}| + 6.4|\mathbf{B}| - 54 \leq 0 && (3) \\ & R_1 - R_2 + \tfrac{1}{2}(d_1 + d_2) < 0 && (4) \end{aligned}$$

where $E_{ref} = 180MJ$, $B_{norm} = 200\mu T$ and x are the design variables $x = (R_1, R_2, h_1/2, h_2/2, d_1, d_2, J_1, J_2)$.

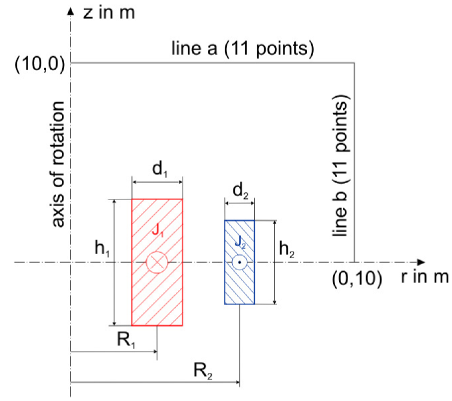

Fig. 2. SMES Device [13]

The first constraint aims to limit the maximal flux density ($|\mathbf{B}|$) in the coils, the maximal values of $|\mathbf{B}|$ are located on their boundaries on coordinates $P_1 = (R_1 - d_1/2, 0)$, $P_2 = (R_1 + d_1/2, 0)$ and $P_3 = (R_2 - d_2/2, 0)$. Thus this constraint is replaced by three constraints (6),(7) and (8).

The second constraint aims to prevent both coils from overlapping. Unfortunately, optimization algorithms can sometimes violate the constraints, this implies taking some special care of this issue that depends on the type of the algorithm used. As we are comparing different approaches, we chose to define another optimization problem equivalent to the initial one that avoids the aforementioned issue. We define a variable A_2 as

$$A_2 = R_2 - R_1 - (d_1 + d_2)/2$$

this new variable is defined in the interval $[0,\ 3.9]$ and replaces the second hard constraint by two soft constraints (9) and (10). Thus, the optimization problem becomes as follows

$$\begin{aligned} \min_x \quad & OF(x) = B^2_{stray}(x)/B^2_{norm} + |E(x) - E_{ref}|/E_{ref} && (5) \\ s.t. \quad & (|\mathbf{J}(x, P_1)| - 54)/6.4 + |\mathbf{B}(x, P_1)| \leq 0 && (6) \\ & (|\mathbf{J}(x, P_2)| - 54)/6.4 + |\mathbf{B}(x, P_2)| \leq 0 && (7) \\ & (|\mathbf{J}(x, P_3)| - 54)/6.4 + |\mathbf{B}(x, P_3)| \leq 0 && (8) \\ & R_1 + A_2 + (d_1 + d_2)/2 \leq 5 && (9) \\ & -R_1 - A_2 - (d_1 + d_2)/2 \leq -1.8 && (10) \end{aligned}$$

where x are the design variables $x = (R_1, A_2, h_1/2, h_2/2, d_1, d_2, J_1, J_2)$. The algorithms setups are as follows

- **Gradient based** : SQP algorithm from Matlab is used with 100 uniformly distributed initial points.
- **GA**: 50 runs are performed, the population size is set 200, selection method is stochastic uniform, crossover method and fraction are scattered and 80% respectively, elite fraction is 5% and mutation method is Gaussian. The constraints are handled by the penalty method.
- **DIRECT**: Jones factor were set to 10^{-4} while increasing the number of iterations [2]. The constraints are handled by penalty method
- **B2M2EP**: we choose to build each meta-model using 20 initial samples generated by latin hypercube sampling and enrich each one of them by 30 samples added using the

prediction error infill criterion and 5 using the expected improvement criterion [11].

TABLE II
TEAM WORKSHOP PROBLEM 25 OPTIMIZATIONS RESULTS

Approach	SQP	GA	DIRECT	B2M2EP
R_1	1.336	1.457	1.543	1.292
A_2	0.027	0.481	0.229	0.058
$h_1/2$	1.011	1.209	0.951	0.95
$h_2/2$	1.452	1.8	1.526	1.481
d_1	0.677	0.347	0.374	0.668
d_2	0.269	0.121	0.217	0.231
J_1	15.579	19.834	22.346	15.338
J_2	-15.069	-17.305	-13.441	-15.000
OF	0.00197	0.03502	0.04881	0.159
# FEM evals	1902	160201	421995	2261685
C.R.	1 %	2 %	100 %	100 %
Expected # FEM evals	529640	94276389	421995	2261685

Here, in TABLE II, we notice the gradient based approach is largely outperforming the other approaches in term solution quality, actually, 17% of the solutions found by this approach has an objective function value less than 0.05 while being distinct in term of the variables, this explain the low C.R. of the method and, thus, the high expected number of evaluations.

Genetic algorithm performed better than DIRECT in term of quality of solution but the former somewhat suffer from its inherent randomness and thus the low convergence rate.

The meta-model based approach was unable to get a good solution in a reasonable computational time, the number of evaluation exceeds 2 millions evaluations. This is due to the number of spaces generated after each splitting, *i.e* 256. Nevertheless, it worth noting that the spaces containing the solutions of the other algorithms are listed as candidate, which means, if the algorithm goes further, it will eventually reach one of these solutions.

VI. CONCLUSION

This article presents a comparison between gradient-based and gradient-free approaches for the optimization of electromagnetic devices using FEM. We treated two known benchmarks from the literature [13][12]. Then we used two metrics for the comparison, the first one is the quality of the solution and the second is the computational cost. The choice of the algorithm from each category is based on what we can find generally in the literature, the choice of a certain algorithm or implementation was based on what we are working on, *i.e.* gradient based and meta-model approaches, and the availability and simplicity of usage. These are usually the challenges that designers are facing when doing optimization.

In term of performances the gradient based approach outperform other strategies for both test cases, this was possible due to the computation of the gradient using the adjoint variable method. This improves drastically the convergence and the quality of the solutions, but this come with the expense of intrusive manipulation of the FEM code.

The meta-model approach remains a good alternative in term of implementation and results quality but can be expensive, the developed approach was able to overcome some on the very known issues when using meta-models for optimization.

Genetic algorithm is one of the most used algorithms to deal with noisy data, for both test cases GA did not perform well. These performances could be slightly improved by doing some parameter tuning or using other implementations.

Direct performed very well for the first test case but it was ineffective for the second one, this can be explained by the constraints handling strategy, the penalty method implemented in the algorithm may not be the best.

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