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Economic and Environmental Decomposition of Luenberger-Hicks-Moorsteen Total Factor Productivity Indicator: Empirical Analysis of Textile Firms with a Focus on Reporting Infeasibilities and Questioning Convexity

Tomas Baležentis, Kristiaan Kerstens, and Zhiyang Shen*

Abstract—We discuss an environmental Luenberger-Hicks-Moorsteen (LHM) Total Factor Productivity (TFP) indicator and its decomposition that incorporates a negative externality into the measurement of economic performance. Special cases of a generalized environmental directional distance function are involved in the definition of this LHM indicator and its proposed decomposition. We also seek to test whether changes in the convexity assumption provoke differences in the TFP measures. We apply two specifications of the by-production nonparametric environmental technology to implement this LHM TFP. This LHM TFP indicator decomposes into three terms representing technical change, technical inefficiency change, and scale inefficiency change. The changes in the environmental TFP for China's textile industry is then estimated for the period from 2001 to 2010. We report infeasibilities and we show the differences of the proposed framework for the decomposition of the LHM indicator depending on the convexity assumption. The results suggest there has been an increase in the TFP of China's textile industry: the amount depends on the convexity or not of the technology. The environmental performance is poorer than the economic one. Moreover, contradictions between convex and nonconvex LHM indicators for individual observations appear for a substantial part of the sample.

Index Terms—Total Factor Productivity, Indicator, Data Envelopment Analysis, Environmental Production Technology.

I. INTRODUCTION

THE analysis of primal total factor productivity (TFP) is important to identify the best practice and the underlying sources of productivity change using knowledge on technologies. Therefore, different TFP indices (using ratios) and indicators (using differences) have been proposed to address the issue. For instance, the Malmquist productivity

index [1] is probably among the most widely used recent productivity measures. Yet [2] argues forcefully that it does not meet the property of completeness (neither in a multiplicative sense using ratios, nor in an additive sense using differences). Therefore, the Malmquist productivity index fails to be a TFP index. Furthermore, [2] states that the Hicks-Moorsteen TFP index proposed by [3] is one of a few indices satisfying the property of completeness.

Since the ratio-based Hicks-Moorsteen index does not allow for zero values of inputs or outputs, the Luenberger-Hicks-Moorsteen (LHM) TFP indicator has been proposed by [4]. The latter indicator improves on the more popular Luenberger productivity indicator [5] that itself fails additive completeness and thus fails to be a TFP indicator. Therefore, the LHM TFP indicator has the following two appealing features: (i) additive completeness (i.e., it serves as a TFP measure), and (ii) additive decomposition (i.e., ability to handle zero values).

While these productivity indices and indicators have been estimated using traditional parametric specifications of technologies (e.g., [6]), the vast bulk of the literature has opted for a nonparametric approach. This allows to analyze the dynamics of productivity change solely based on technology information and without resorting to data on input and output prices.¹

Given the environmental considerations raised by such international bodies as the United Nations [7,8], the measurement of green efficiency and productivity growth has become a very topical issue. Therefore, an important effort has been done to extend the measures of productive efficiency and productivity to account for a variety of environmental pressures (see the surveys by [9-11]). One of the major concerns underpinning the green productivity growth is ensuring energy

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¹ In the operations research literature, these nonparametric production technology models go under the name Data Envelopment Analysis (moniker DEA).

decoupling and dematerialization [12]. Inclusion of the energy input in the efficiency and productivity analysis framework allows assessing the energy performance from an integrated perspective based on neoclassical economic theory; see, e.g., [13-15].

There are nowadays quite some empirical applications of the LHM TFP indicator: a recent example includes, for instance, [16]. But, it is clear that the LHM TFP indicator is nowhere as popular as the Luenberger productivity indicator: a Google Scholar search on 12 April 2022 obtained 187 results for the expression “Luenberger-Hicks-Moorsteen productivity”, while a search for “Luenberger productivity” yields 2510 hits. [18] seem to be the first to systematically compare the Luenberger and the LHM TFP indicators. [20] show that the Bennet indicator is a superlative indicator for the LHM indicator under certain conditions.

[19] are to the best of our knowledge the first to apply the LHM TFP indicator to measure green or environmental productivity. However, these authors model the undesirable outputs in the same manner as the desirable outputs and use the resulting distance functions along with those based on the technology involving no undesirable outputs at all. Abad [20] proposes an environmental generalized LHM TFP indicator which is based on directional distance functions involving either the reduction of inputs and undesirable outputs, or the expansion of desirable outputs only. Therefore, the undesirable outputs are essentially treated as inputs.

We depart from this setting by focusing on the optimization of inputs and all kinds of outputs separately by means of respective directional distance functions. We also assume only costly disposability of the undesirable outputs. Furthermore, we propose a decomposition of the environmental LHM indicator allowing one to consider the three terms of technical change, technical inefficiency change, and scale inefficiency change proposed in [21,22].

The proposed approach relies on the LHM TFP indicator as defined by [4]. We then extend the indicator following [20] to accommodate the undesirable outputs and we propose a decomposition of the environmental LHM indicator in line with [21,22] and [23]. However, we suggest modeling the by-production technology as proposed by [24] and as further elaborated upon by [25]. This by-production technology maintains costly disposability of the undesirable outputs.

We extend [24] as well as [25] by also allowing for a nonconvex technology. While it is well-known that environmental externalities create nonconvexities in the technology of the firms affected by the externalities, it is surprising to realise that almost all of the economic literature ignores these nonconvexities when modelling production with environmental externalities. Furthermore, apart from environmental externalities, there can also be other sources for nonconvexities in production: indivisibilities in inputs and outputs, economies of scale, and economies of specialization, among others. Therefore, the traditional assumption of convexity maintained in almost all of the economic literature needs to be scrutinized.

Seemingly, somehow economists seem to suppose that

convex models provide an acceptable approximation to a nonconvex production reality. However, this acceptable approximation is not guaranteed when analysing technologies. For instance, [17] show that technology-based LHM TFP indicators differ substantially under convexity and nonconvexity. Furthermore, convex models need not provide a good approximation in the case of economic value functions (like, e.g., the cost function). For instance, [26] illustrate that the gap between convex and nonconvex cost function levels may be very substantial. Furthermore, these same authors show that this may result in contradictory results for returns to scale as well as for economies of scale for a substantial part of the sample.

Within the by-production approach adopted in this contribution, we are only aware of [27] who also provide a nonconvex perspective in addition to a traditional convex one at the level of efficiency measurement. In this contribution, we offer to the best of our knowledge for the first time such a complementary convex and nonconvex perspective at the level of the LHM TFP indicator. An earlier comparison of convex and nonconvex LHM TFP indicators is found in [17], but these authors focus on traditional production and not environmental production.

Even though a major part of the engineering literature draws on some form of nonconvex and/or nonlinear optimization models rather than on basic convex optimization models, in engineering management one sometimes uncritically adopts convex models borrowed from the production economics literature. For instance, the optimal power flow problem in its generality is mixed-integer linear or nonlinear; see [28]. It is rather easy to find engineering applications that assess the efficiency of electricity generation (e.g., [29]) or electricity distribution (e.g., [30]) using convex production models. However, [31] argue and empirically illustrate that the specification of convex or nonconvex technologies impacts the measurement of the efficiency of electricity distributors. Thus, it is essential to clearly document the impact of convexity on modelling production relations. When convex models provide a poor approximation to the results of the nonconvex models, then it is natural to opt for the more plausible hypothesis of nonconvexity and remove the simplifying convexity assumption.

The LHM TFP indicator is applied for a sample of Chinese textile companies. Since the Chinese economy continues expanding, there is a need for establishing proper mechanisms to support the sustainability of its sectors. Thus, the application of the LHM TFP indicator allows one to ascertain if the economic and environmental performance has improved over time. Also, the decomposition into the sources of TFP and the contributions of the economic and environmental dimensions further allows the policy makers to draw reasonable policy guidelines. To the best of our knowledge the LHM productivity indicator has not been applied to assess the green TFP change in the Chinese textile sector.

In brief, we try to achieve in this contribution the following goals. First, while we are not the first empirical application of the LHM productivity indicator focusing on undesirable outputs

using the [24] specification of the costly disposable technology and applying the [21,22] and the [23] decomposition, we are unaware of any other contributions focusing on the Chinese textile sector. Furthermore, no other contribution contrasted the [24] and the [25] specifications of the by-production approach using both convex and nonconvex technologies. Second, while it is well-known that the Hicks-Moorsteen TFP index can always be computed under weak conditions on technology (see [32]), the environmental technology with costly disposal in the undesirable outputs can lead to infeasibilities. We are to the best of our knowledge the first to explore to which extent an environmental LHM TFP indicator also suffers from a lack of determinateness under convex and nonconvex settings.

This contribution is structured as follows. Section II presents the methodology for the analysis of the environmental TFP change. More specifically, the environmental production technology, directional distance functions with corresponding estimators, and the decomposition of the environmental LHM indicator are discussed. Section III brings together the empirical results of the application of the proposed environmental indicator to the sample of the Chinese textile companies. We compare the results based on the convex and the nonconvex technologies and discuss the patterns in the environmental TFP change prevailing among the companies analysed. Finally, Section IV concludes.

II. METHODOLOGY

This section presents the methodology for the proposed decomposition of the LHM TFP indicator. First, the environmental technology and the generalized environmental directional distance functions are discussed. Second, we focus on the decomposition of the LHM indicator. Third, the nonparametric technologies satisfying the desirable axioms are presented.

A. Environmental Production Technology and Directional Distance Function

We follow a multiple-input multiple-output approach involving both a vector of desirable and a vector of undesirable outputs. Assume that each decision-making unit has $N+M$ inputs (\mathbf{x}), G desirable outputs (\mathbf{y}), and P undesirable outputs (\mathbf{z}). Following [24], we can define the environmental production possibility set at the time period t as follows:

$$\begin{aligned} T(t) &= T_{eco}(t) \cap T_{env}(t) \\ T_{eco}(t) &= \left\{ (\mathbf{x}^t, \mathbf{y}^t) \in \mathbb{R}_+^{N+M+G} : \mathbf{x}^t \text{ can produce } \mathbf{y}^t \right\}; \\ T_{env}(t) &= \left\{ (\mathbf{x}^t, \mathbf{z}^t) \in \mathbb{R}_+^{M+P} : \mathbf{x}^t \text{ can generate } \mathbf{z}^t \right\}. \end{aligned} \quad (1)$$

where the production technology (T) can be separated into two sub-technologies. The first sub-technology (T_{eco}) is the conventional economic technology which assumes traditional axioms, such as no free lunch, convexity, variable returns to scale (VRS), and monotonicity, among others. The second sub-technology (T_{env}) is the sub-frontier modeling environmental production technology under costly disposability, convexity, variable returns (damage) to scale. The two sub-technologies

are bounded (finite). A detailed discussion and illustration of the environmental axioms for the by-production technology is available in [24]. Note that convexity is not always maintained on both sub-technologies, since it can only be interpreted in terms of perfect time divisibility and this assumption is questionable in technologies.

All the inputs ($N+M$) can be divided into those that produce desirable outputs only (N inputs), and those that can generate undesirable outputs (M inputs). The latter inputs can be regarded as pollution-generating inputs. From the economic point of view, good outputs bring benefits for social welfare and thus need to be increased, while bad outputs generate negative externalities and therefore need to be reduced. Obviously, also inputs are scarce and ought to be reduced. This environmental production technology can be represented by the directional distance function following [33] and [34]. A generalized directional distance function simultaneously defining an increase in desirable outputs and a contraction in undesirable outputs as well as in inputs for period $a \in \{t, t+1\}$ with respect to a technology in period $b \in \{t, t+1\}$ can be defined as:

$$D^b(\mathbf{x}^a, \mathbf{y}^a, \mathbf{z}^a; \mathbf{g}_x^a, \mathbf{g}_y^a, \mathbf{g}_z^a) = \max \{ \delta_{inp}, \delta_{eco}, \delta_{env} \in \mathbb{R}_+ : (\mathbf{x}^a - \delta_{inp} \mathbf{g}_x^a, \mathbf{y}^a + \delta_{eco} \mathbf{g}_y^a, \mathbf{z}^a - \delta_{env} \mathbf{g}_z^a) \in T(t) \} \quad (2)$$

where $(\mathbf{g}_x^t, \mathbf{g}_y^t, \mathbf{g}_z^t) \geq \mathbf{0}$ are directional vectors of inputs, desirable and undesirable outputs. Three scalars measure the maximum possible increase in desirable outputs (δ_{eco}) and the decrease in undesirable outputs (δ_{env}) and inputs (δ_{inp}), and the notation $(a, b) \in \{t, t+1\} \times \{t, t+1\}$ allows for the mixed-period directional distance functions.

B. Environmental LHM Indicator and A Novel Decomposition

An Environmental LHM Indicator. [4] define the LHM productivity indicator which can be regarded as an additively complete TFP indicator following the definition by [2]. The main objective of this contribution is to extend the LHM indicator by incorporating the undesirable outputs into the analysis. By doing so, we can offer an approach for the analysis of an environmentally adjusted TFP indicator.

There are several possibilities for incorporating the undesirable outputs into a productivity or a TFP measure [9-11]: the undesirable outputs can be regarded as inputs and reduced with inputs simultaneously; they can enter the model as weakly disposable outputs; or a costly disposability of the undesirable outputs may be assumed, among others. We follow the latter approach and opt for increasing the desirable outputs and reducing the undesirable ones simultaneously during the optimization within the by-production technology.

The environmental LHM indicator measures the change in the environmental TFP by considering the distances between the frontier and observations for periods t and $t+1$. This is done along the direction of (desirable and undesirable) outputs

while keeping input quantities fixed at the base period, and along the direction of inputs while keeping output levels fixed at the base period. To avoid arbitrariness when choosing the base period, the measures are implemented by treating each of the two periods in turn as the base period.

We define the environmental LHM indicator for the base period t as follows:

$$LHM^t = [D'(x_k^t, y_k^t, z_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t) - D'(x_k^t, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})] - [D'(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{0}) - D'(x_k^t, y_k^t, z_k^t; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0})] \quad (3)$$

where the first two terms in the brackets capture the distance to the frontier of period t along the direction of desirable and undesirable outputs, whereas the last two terms capture the distance to the frontier along the direction of inputs. Whenever this indicator is higher (resp. lower) than zero, then we observe an environmental TFP gain (resp. loss). Similarly, we define the LHM indicator for the base period $t+1$ as follows:

$$LHM^{t+1} = [D^{t+1}(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1}) - D^{t+1}(x_k^t, y_k^t, z_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t)] - [D^{t+1}(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{0}) - D^{t+1}(x_k^t, y_k^t, z_k^t; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0})] \quad (4)$$

Then, by taking the arithmetic average of the period t and $t+1$ indicators given in (3) and (4), respectively, one arrives at the LHM productivity change indicator between periods t and $t+1$:

$$LHM^{t,t+1} = \frac{1}{2}(LHM^t + LHM^{t+1}) = \frac{1}{2} \left(\begin{aligned} & [D'(x_k^t, y_k^t, z_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t) - D'(x_k^t, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})] \\ & - [D'(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{0}) - D'(x_k^t, y_k^t, z_k^t; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0})] \\ & + [D^{t+1}(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1}) - D^{t+1}(x_k^t, y_k^t, z_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t)] \\ & - [D^{t+1}(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{0}) - D^{t+1}(x_k^t, y_k^t, z_k^t; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0})] \end{aligned} \right) \quad (5)$$

A Novel Decomposition for Green TFP indicator. According to [21,22] and the empirical application in [23], we can decompose the environmental LHM indicator using the output direction (output side) or using the input direction (input side) into the following three components:

$$LHM^{t,t+1} = TEC^{t,t+1} + TP^{t,t+1} + SEC^{t,t+1} \quad (6)$$

where TEC is technical inefficiency change, TP is technological progress/regress, and SEC is scale inefficiency change.

In this contribution, we opt for the output direction to decompose the TFP: this is denoted by the subscript “output”. First, the TEC component captures the change in resource utilization as compared to a contemporaneous frontier:

$$TEC_{output}^{t,t+1} = D'(x_k^t, y_k^t, z_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t) - D^{t+1}(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1}) \quad (7)$$

with $TEC_{output}^{t,t+1} > 0$ (resp. $TEC_{output}^{t,t+1} < 0$) indicating gains (resp. losses) in the environmental TFP due to DMU-specific improvements (resp. deterioration) in their activities. Basically, in the case that a positive value of TEC is observed, this term indicates the extent of increase in the desirable outputs and decrease in the undesirable ones by keeping the input level fixed resulting in an improved performance of a DMU.

Second, from the output side, the TP component indicates the productivity gain due to technological innovation and it is computed as follows:

$$TP_{output}^{t,t+1} = \frac{1}{2} \left([D^{t+1}(x_k^t, y_k^t, z_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t) - D'(x_k^t, y_k^t, z_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t)] + [D^{t+1}(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1}) - D'(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})] \right) \quad (8)$$

where the first two terms measure the shift in the frontier from period t to period $t+1$ with respect to the observation from period t , whereas the last two terms measure the same shift with respect to the observation from period $t+1$. When $TP^{t,t+1} > 0$ (resp. $TP^{t,t+1} < 0$), then this component indicates technical progress (resp. regress).

Finally, the SEC component shows the additive residual and indicates whether the evaluated production plan is getting closer to or further away from the most productive scale size as represented by the change in the gradient of the frontier:

$$LHM_{output}^{t,t+1} - TEC_{output}^{t,t+1} - TP_{output}^{t,t+1} = \frac{1}{2} \left(\begin{aligned} & [D'(x_k^t, y_k^t, z_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t) - D'(x_k^t, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})] \\ & - [D'(x_k^t, y_k^t, z_k^t; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0}) - D'(x_k^t, y_k^t, z_k^t; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0})] \\ & + [D^{t+1}(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1}) - D^{t+1}(x_k^t, y_k^t, z_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t)] \\ & - [D^{t+1}(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{0}) - D^{t+1}(x_k^t, y_k^t, z_k^t; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0})] \end{aligned} \right) \quad (9)$$

where the first four terms measure the gradient of the frontier for period t in the region spanned by x^t and x^{t+1} , whereas the last four terms measure the gradient of the frontier for period $t+1$ in the same region.

Following [22] and [23], this expression (9) can be rewritten by using the translation property of the directional distance function:

$$SEC_{output}^{t,t+1} = \frac{1}{2} \left(\begin{aligned} & [D'(x_k^t, y_k^t, z_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t) - D'(x_k^t, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})] \\ & - [D'(x_k^t, y_k^t, z_k^t; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0}) - D'(x_k^t, y_k^t, z_k^t; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0})] \\ & + \frac{1}{2} \left([D^{t+1}(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1}) - D^{t+1}(x_k^t, y_k^t, z_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t)] \right. \\ & \left. - [D^{t+1}(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{0}) - D^{t+1}(x_k^t, y_k^t, z_k^t; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0})] \right) \end{aligned} \right) \quad (10)$$

whereby:

$$(y_k^{t,*}, z_k^{t,*}) = (y_k^t, z_k^t) + D'(x_k^t, y_k^t, z_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t)(\mathbf{g}_y^t, \mathbf{g}_z^t) \quad (11)$$

$$(y_k^{t+1,**}, z_k^{t+1,**}) = (y_k^{t+1}, z_k^{t+1}) + D'(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})(\mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})$$

and

$$(y_k^{t,**}, z_k^{t,**}) = (y_k^t, z_k^t) + D^{t+1}(x_k^t, y_k^t, z_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t)(\mathbf{g}_y^t, \mathbf{g}_z^t) \quad (12)$$

$$(y_k^{t+1,*}, z_k^{t+1,*}) = (y_k^{t+1}, z_k^{t+1}) + D^{t+1}(x_k^{t+1}, y_k^{t+1}, z_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})(\mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})$$

Note that expression (11) defines the efficient values of outputs, $(y_k^{t,*}, z_k^{t,*})$ and $(y_k^{t+1,**}, z_k^{t+1,**})$, for respective levels of input use at different time periods with respect to a technology of period t . Similarly, expression (12) defines the optimal output levels, $(y_k^{t,**}, z_k^{t,**})$ and $(y_k^{t+1,*}, z_k^{t+1,*})$, relative to a technology of period $t+1$. Therefore, expressions (11) and (12) define the dynamics in the shape of the frontiers as represented by their efficient points.

The directional distance function contains three sub-scores for measuring possible inputs decrease (δ_{inp}), potential desirable outputs expansion (δ_{eco}) and undesirable outputs reduction (δ_{env}). The LHM indicator can be separated into an economic ($LHM_{eco}^{t,t+1}$) and an environmental component ($LHM_{env}^{t,t+1}$) as follows:

$$\begin{aligned} LHM^{t,t+1} &= LHM_{eco}^{t,t+1} + LHM_{env}^{t,t+1} \\ &= TEC_{eco}^{t,t+1} + TP_{eco}^{t,t+1} + SEC_{eco}^{t,t+1} + TEC_{env}^{t,t+1} + TP_{env}^{t,t+1} + SEC_{env}^{t,t+1} \end{aligned} \quad , \quad (13)$$

where economic and environmental TFP gains can be further decomposed into TEC, TP and SEC elements.

C. Estimation Strategy

The directional distance function can be estimated by employing parametric or nonparametric approaches. We opt for the nonparametric approach which allows for the estimation of the production frontier without specifying any specific functional form and which imposes a minimum amount of a priori assumptions like monotonicity and convexity on the technology if needed. Note that convexity is not always maintained in this contribution.

Following [24], we can define the convex nonparametric environmental production technology, $\hat{T}_C^M(t)$, as follows:

$$\hat{T}_C^M(t) = \{(\mathbf{x}^t, \mathbf{y}^t, \mathbf{z}^t) \in \mathbb{R}_+^{N+M+G+P} : \begin{aligned} &\sum_{k=1}^K \lambda_k y_k^{g,t} \geq y^{g,t}, \quad g=1, \dots, G; \\ &\sum_{k=1}^K \lambda_k x_k^{n,t} \leq x^{n,t}, \quad n=1, \dots, N; \\ &\sum_{k=1}^K \lambda_k x_k^{m,t} \leq x^{m,t}, \quad m=1, \dots, M; \\ &\sum_{k=1}^K \mu_k z_k^{p,t} \leq z^{p,t}, \quad p=1, \dots, P; \\ &\sum_{k=1}^K \mu_k x_k^{m,t} \geq x^{m,t}, \quad m=1, \dots, M; \\ &\sum_{k=1}^K \lambda_k = 1, \sum_{k=1}^K \mu_k = 1; \\ &\lambda_k \geq 0, \quad \mu_k \geq 0, k=1, \dots, K \end{aligned} \quad , \quad (14A)$$

where λ_k and μ_k are activity variables for the two sub-frontiers. Similarly, the nonconvex by-production technology, $\hat{T}_{NC}^M(t)$, is defined as:

$$\hat{T}_{NC}^M(t) = \{(\mathbf{x}^t, \mathbf{y}^t, \mathbf{z}^t) \in \mathbb{R}_+^{N+M+G+P} : \begin{aligned} &\sum_{k=1}^K \lambda_k y_k^{g,t} \geq y^{g,t}, \quad g=1, \dots, G; \\ &\sum_{k=1}^K \lambda_k x_k^{n,t} \leq x^{n,t}, \quad n=1, \dots, N; \\ &\sum_{k=1}^K \lambda_k x_k^{m,t} \leq x^{m,t}, \quad m=1, \dots, M; \\ &\sum_{k=1}^K \mu_k z_k^{p,t} \leq z^{p,t}, \quad p=1, \dots, P; \\ &\sum_{k=1}^K \mu_k x_k^{m,t} \geq x^{m,t}, \quad m=1, \dots, M; \\ &\sum_{k=1}^K \lambda_k = 1, \sum_{k=1}^K \mu_k = 1; \\ &\lambda_k \in \{0,1\}, \quad \mu_k \in \{0,1\}, k=1, \dots, K \end{aligned} \quad (14B)$$

where again λ_k and μ_k are activity variables for the two sub-frontiers.

[25] propose an improved by-production model linking two sub-technologies. In the fifth constraint, instead of regarding the pollution-generating inputs as “outputs” (

$$\sum_{k=1}^K \mu_k x_k^{m,t} \geq x^{m,t})^2 \text{ in the environmental sub-technology,}$$

² The inequality in the fifth constraint of Eqs. 14A and 14B implies that the pollution-generating inputs are considered as a kind of outputs in this sub-technology.

[25] argue that the optimal quantity use of pollution-generating inputs should be identical between two sub-technologies ($\sum_{k=1}^K \mu_k x_k^{m,t} = \sum_{k=1}^K \lambda_k x_k^{m,t}$). This new constraint allows for a linkage between sub-technologies. Therefore, we can define the convex nonparametric environmental production technology, $\hat{T}_C^B(t)$, as follows:

$$\hat{T}_C^B(t) = \{(\mathbf{x}^t, \mathbf{y}^t, \mathbf{z}^t) \in \mathbb{R}_+^{N+M+G+P} : \begin{aligned} &\sum_{k=1}^K \lambda_k y_k^{g,t} \geq y^{g,t}, \quad g=1, \dots, G; \\ &\sum_{k=1}^K \lambda_k x_k^{n,t} \leq x^{n,t}, \quad n=1, \dots, N; \\ &\sum_{k=1}^K \lambda_k x_k^{m,t} \leq x^{m,t}, \quad m=1, \dots, M; \\ &\sum_{k=1}^K \mu_k z_k^{p,t} \leq z^{p,t}, \quad p=1, \dots, P; \\ &\sum_{k=1}^K \mu_k x_k^{m,t} = \sum_{k=1}^K \lambda_k x_k^{m,t}; \\ &\sum_{k=1}^K \lambda_k = 1, \sum_{k=1}^K \mu_k = 1; \\ &\lambda_k \geq 0, \quad \mu_k \geq 0, k=1, \dots, K \end{aligned} \quad (15A)$$

Similarly, the nonconvex by-production technology, $\hat{T}_{NC}^B(t)$, of [25] is defined as:

$$\hat{T}_{NC}^B(t) = \{(\mathbf{x}^t, \mathbf{y}^t, \mathbf{z}^t) \in \mathbb{R}_+^{N+M+G+P} : \begin{aligned} &\sum_{k=1}^K \lambda_k y_k^{g,t} \geq y^{g,t}, \quad g=1, \dots, G; \\ &\sum_{k=1}^K \lambda_k x_k^{n,t} \leq x^{n,t}, \quad n=1, \dots, N; \\ &\sum_{k=1}^K \lambda_k x_k^{m,t} \leq x^{m,t}, \quad m=1, \dots, M; \\ &\sum_{k=1}^K \mu_k z_k^{p,t} \leq z^{p,t}, \quad p=1, \dots, P; \\ &\sum_{k=1}^K \mu_k x_k^{m,t} = \sum_{k=1}^K \lambda_k x_k^{m,t}; \\ &\sum_{k=1}^K \lambda_k = 1, \sum_{k=1}^K \mu_k = 1; \\ &\lambda_k \in \{0,1\}, \quad \mu_k \in \{0,1\}, k=1, \dots, K \end{aligned} \quad (15B)$$

To calculate the LHM indicator in expressions (4) and (5) and its components, one needs to solve a series of mathematical programs. Here, we present only the two particular cases where input-output vectors from period $a \in \{t, t+1\}$ are compared against a technology of period $b \in \{t, t+1\}$ as defined by the corresponding output or input directional distance functions. Let us assume that there are K decision making units (DMUs) indexed by $k = 1, 2, \dots, K$. The input-output vectors of these units then serve to construct an empirical frontier. Taking the nonconvex approach in the [25] specification as an example, the output directional distance function $D^b(\mathbf{x}^a, \mathbf{y}^a, \mathbf{z}^a; \mathbf{0}, \mathbf{g}_y^a, \mathbf{g}_z^a)$ is obtained via solving the following binary mixed integer linear program (BMILP1):

$$\begin{aligned}
D^b(\mathbf{x}^a, \mathbf{y}^a, \mathbf{z}^a; \mathbf{0}, \mathbf{g}_y^a, \mathbf{g}_z^a) &= \max_{\delta, \lambda, \sigma} \frac{1}{2} (\delta_{eco} + \delta_{env}) \\
s.t. \quad \sum_{k=1}^K \lambda_k y_k^{g,b} &\geq y^{g,a} + \delta_{eco} g_y^{g,a}, \quad g=1, \dots, G; \\
\sum_{k=1}^K \lambda_k x_k^{n,b} &\leq x^{n,a}, \quad n=1, \dots, N; \\
\sum_{k=1}^K \lambda_k x_k^{m,b} &\leq x^{m,a}, \quad m=1, \dots, M; \\
\sum_{k=1}^K \lambda_k &= 1; \\
\lambda_k &\in \{0, 1\}, \quad k=1, \dots, K; \\
\sum_{k=1}^K \mu_k z_k^{p,b} &\leq z^{p,a} - \delta_{env} g_z^{p,a}, \quad p=1, \dots, P; \\
\sum_{k=1}^K \mu_k x_k^{m,b} &= \sum_{k=1}^K \lambda_k x_k^{m,b} \\
\sum_{k=1}^K \mu_k &= 1; \\
\mu_k &\in \{0, 1\}, \quad k=1, \dots, K,
\end{aligned} \tag{BMILP1}$$

where δ_{eco} and δ_{env} are the weighted value of the output directional distance function showing maximum expansions in good outputs and reductions in bad outputs for the direction vector defined by $(\mathbf{0}, \mathbf{g}_y^a, \mathbf{g}_z^a)$. Again for the nonconvex approach in the Baležentis et al. [25] specification, the input directional distance function $D^b(\mathbf{x}^a, \mathbf{y}^a, \mathbf{z}^a; \mathbf{g}_x^a, \mathbf{0}, \mathbf{0})$ is obtained via solving the following linear program (BMILP2):

$$\begin{aligned}
D^b(\mathbf{x}^a, \mathbf{y}^a, \mathbf{z}^a; \mathbf{g}_x^a, \mathbf{0}, \mathbf{0}) &= \max_{\delta, \lambda, \sigma} \delta_{inp} \\
s.t. \quad \sum_{k=1}^K \lambda_k y_k^{g,b} &\geq y^{g,a}, \quad g=1, \dots, G; \\
\sum_{k=1}^K \lambda_k x_k^{n,b} &\leq x^{n,a} - \delta_i g_x^{n,a}, \quad n=1, \dots, N; \\
\sum_{k=1}^K \lambda_k x_k^{m,b} &\leq x^{m,a} - \delta_i g_x^{m,a}, \quad m=1, \dots, M; \\
\sum_{k=1}^K \lambda_k &= 1; \\
\lambda_k &\in \{0, 1\}, \quad k=1, \dots, K; \\
\sum_{k=1}^K \mu_k z_k^{p,b} &\leq z^{p,a}, \quad p=1, \dots, P; \\
\sum_{k=1}^K \mu_k x_k^{m,b} &= \sum_{k=1}^K \lambda_k x_k^{m,b} \\
\sum_{k=1}^K \mu_k &= 1; \\
\mu_k &\in \{0, 1\}, \quad k=1, \dots, K;
\end{aligned} \tag{BMILP2}$$

where δ_{inp} is the value of the input directional distance function denoting the maximum contraction in inputs for the direction vector defined by $(\mathbf{g}_x^a, \mathbf{0}, \mathbf{0})$ at period $a \in \{t, t+1\}$. Note that the estimation of the LHM indicator also requires

mixing the periods of input and output vectors in certain instances, yet these calculations are straightforward generalizations of the mathematical programming models given above. The mathematical programming problems for the [24] specification are very similar and are left to the reader.

Full efficiency is represented by zero values of the directional distance function, whereas positive values indicate inefficiency. In practice, the direction vector \mathbf{g} is selected as equaling the components of inputs and outputs of the evaluated DMUs. Therefore, the optimal efficiency scores have a proportional interpretation and are expressed as a percentage of the chosen direction vectors. For instance, if $\delta_{inp} = 2\%$, then this implies that the firm should be capable to reduce all of its inputs by 2%. [35] show that the proportional distance function (PDF) satisfies strong commensurability or unit invariance while the directional distance function does not satisfy this essential property.

Let $\delta \mid D^a(\mathbf{x}_k^b, \mathbf{y}_k^b, \mathbf{z}_k^b; \mathbf{g}_x^b, \mathbf{g}_y^b, \mathbf{g}_z^b)$ denote a solution of a certain mathematical programming problem as described above, where $\delta \in \{\delta_{eco}, \delta_{env}, \delta_i\}$. Then, due to the additive nature of the objective function in (BMILP1), the TFP growth can be decomposed into the economic and environmental terms, i.e., $LHM^{t,t+1} = LHM_{eco}^{t,t+1} + LHM_{env}^{t,t+1}$. For a certain observation k , this decomposition can be computed as follows:

$$LHM_{eco}^{t,t+1} = \frac{1}{4} \begin{pmatrix} [\delta_{eco} \mid D(\mathbf{x}_k^t, \mathbf{y}_k^t, \mathbf{z}_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t) - \delta_{eco} \mid D(\mathbf{x}_k^t, \mathbf{y}_k^{t+1}, \mathbf{z}_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})] \\ -[\delta_i \mid D(\mathbf{x}_k^t, \mathbf{y}_k^t, \mathbf{z}_k^t; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0}) - \delta_{inp} \mid D(\mathbf{x}_k^t, \mathbf{y}_k^t, \mathbf{z}_k^t; \mathbf{g}_x^t, \mathbf{g}_y^t, \mathbf{0})] \\ +[\delta_{eco} \mid D(\mathbf{x}_k^{t+1}, \mathbf{y}_k^{t+1}, \mathbf{z}_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1}) - \delta_{eco} \mid D(\mathbf{x}_k^{t+1}, \mathbf{y}_k^{t+1}, \mathbf{z}_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})] \\ -[\delta_{inp} \mid D(\mathbf{x}_k^{t+1}, \mathbf{y}_k^{t+1}, \mathbf{z}_k^{t+1}; \mathbf{g}_x^{t+1}, \mathbf{0}, \mathbf{0}) - \delta_{inp} \mid D(\mathbf{x}_k^{t+1}, \mathbf{y}_k^{t+1}, \mathbf{z}_k^{t+1}; \mathbf{g}_x^{t+1}, \mathbf{g}_y^{t+1}, \mathbf{0})] \end{pmatrix}, \tag{16}$$

$$LHM_{env}^{t,t+1} = \frac{1}{4} \begin{pmatrix} [\delta_{env} \mid D(\mathbf{x}_k^t, \mathbf{y}_k^t, \mathbf{z}_k^t; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t) - \delta_{env} \mid D(\mathbf{x}_k^t, \mathbf{y}_k^t, \mathbf{z}_k^t; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})] \\ -[\delta_i \mid D(\mathbf{x}_k^t, \mathbf{y}_k^t, \mathbf{z}_k^t; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0}) - \delta_i \mid D(\mathbf{x}_k^t, \mathbf{y}_k^t, \mathbf{z}_k^t; \mathbf{g}_x^t, \mathbf{g}_y^t, \mathbf{0})] \\ +[\delta_{env} \mid D(\mathbf{x}_k^{t+1}, \mathbf{y}_k^{t+1}, \mathbf{z}_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1}) - \delta_{env} \mid D(\mathbf{x}_k^{t+1}, \mathbf{y}_k^{t+1}, \mathbf{z}_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})] \\ -[\delta_i \mid D(\mathbf{x}_k^{t+1}, \mathbf{y}_k^{t+1}, \mathbf{z}_k^{t+1}; \mathbf{g}_x^{t+1}, \mathbf{0}, \mathbf{0}) - \delta_i \mid D(\mathbf{x}_k^{t+1}, \mathbf{y}_k^{t+1}, \mathbf{z}_k^{t+1}; \mathbf{g}_x^{t+1}, \mathbf{g}_y^{t+1}, \mathbf{0})] \end{pmatrix}. \tag{17}$$

D. Environmental LHM Indicator and Infeasibility

It is well-known that the Hicks-Moorsteen TFP index can always be computed (determinateness) under weak conditions on the technology [32]: mainly strong disposal of inputs and outputs. [32] conjecture that also the LHM TFP indicator is determinate under the same conditions.

While the environmental technology (1) selected here imposes costly disposability in the undesirable outputs, it is not a traditional technology but an intersection of two sub-technologies. In the literature, there is some scant evidence that an environmental Hicks-Moorsteen TFP index under weak disposability can lead to infeasibilities: see, for instance, the work by [36,37]. However, there has been no studies identifying infeasibilities for the case of the LHM TFP indicator under the by-production technology. In the case of the occurrence of infeasibilities, [38] simply suggest reporting these infeasibilities in detail.

Therefore, it is an open question to which extent an environmental LHM TFP indicator using a by-production

model also suffers from a lack of determinateness. To the best of our knowledge, this issue has not yet been dealt with in the literature.

III. DATA AND EMPIRICAL RESULTS

The proposed methodology is applied on a data set describing production and environmental impacts of a small sample of textile companies in China. This section presents the data employed in detail. Thereafter, the empirical results are discussed.

A. Data

The sample consists of a selection of 56 textile companies in China. The period covered are the years from 2001 to 2010. This yields a total of 560 observations.

We use four inputs, namely labor force, fixed assets, water and energy consumption (the latter is expressed in coal terms). There is one desirable output, gross output value, representing the level of economic activity. In addition, there are three undesirable outputs: waste water, SO₂ emission, and dust emissions. These quantify the environmental pressures of these textile firms. The undesirable outputs are generated by water and energy consumption. The data come from the enterprise database collected by the research group at the School of Management and Economics of the Beijing Institute of Technology.

Table I presents the descriptive statistics for the four inputs and the single good and the three undesirable outputs. Looking at these descriptive statistics suggest that there is a substantial variability in the inputs and outputs at the company level. Therefore, measurement of efficiency and productivity growth is important to ascertain whether these differences in input consumption and output production may be impacted by efficiency and productivity gaps.

TABLE I
DESCRIPTIVE STATISTICS FOR INPUT AND OUTPUTS VARIABLES

Variable	Unit	Mean	Std. Dev.	Min	Max
Labor	Persons	471.1	397.6	101.0	2560.0
Fixed assets	10 ⁶ Yuan	106.3	142.0	11.2	1177.5
Water	10 ³ ton	735.5	779.3	5.0	6928.0
Coal	10 ³ ton	7.1	7.6	0.4	85.0
Gross output	10 ⁷ Yuan	8.9	10.7	1.0	160.0
Waste water	10 ³ ton	521.0	495.2	4.4	3135.1
SO ₂ emission	10 ⁷ m ³	8.9	15.8	0.2	231.6
Dust	ton	28.7	39.0	0.1	486.0

Note: All monetary units are deflated at a constant price level in 2005.

B. Empirical Results

First, the infeasibility issue is documented when measuring two adjacent period directional distance functions as indicated below in Table II. While the total number of observations is 560, each distance function is compared to two time periods yielding 504 results (56 firms × 9 years). We report the infeasibilities both for the [24] and the [25] specifications.

As shown in Table II, the use of the nonconvex technology involves a higher number of infeasibilities in general compared

to the convex technology. In the case of the nonconvex technology, even contemporaneous distance functions may face an infeasibility issue (first and second rows). This seems to happen more rarely for the traditional convex technology. Otherwise, the ranking of the distance functions in terms of the number of infeasibilities remains the same for either convex or nonconvex technologies, i.e., the input-oriented mixed-period distance functions (third row) are the most likely to cause infeasibility. The output-oriented distance functions (fourth and fifth rows) are less affected. Even though the nonconvex technology faces a higher number of infeasibilities, it is still very appealing from the economic engineering viewpoint since it avoids the questionable axiom of convexity. The [25] specification seems less vulnerable to infeasibilities than the original [24] specification.

TABLE II
NUMBER OF INFEASIBILITIES IN NONCONVEX AND CONVEX MEASURES

Model specification	Murty et al. [24]		Baležentis et al. [25]	
	Nonconvex	Convex	Nonconvex	Convex
Technology				
Equation	14B	14A	15B	15A
$D^t(\mathbf{x}_k^t, \mathbf{y}_k^t, \mathbf{z}_k^t; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0})$	6	-	6	-
$D^{t+1}(\mathbf{x}_k^t, \mathbf{y}_k^{t+1}, \mathbf{z}_k^{t+1}; \mathbf{g}_x^t, \mathbf{0}, \mathbf{0})$	284	145	17	-
$D^t(\mathbf{x}_k^{t+1}, \mathbf{y}_k^t, \mathbf{z}_k^t; \mathbf{g}_x^{t+1}, \mathbf{0}, \mathbf{0})$	331	191	16	-
$D^{t+1}(\mathbf{x}_k^{t+1}, \mathbf{y}_k^{t+1}, \mathbf{z}_k^{t+1}; \mathbf{0}, \mathbf{g}_y^t, \mathbf{g}_z^t)$	86	70	73	59
$D^t(\mathbf{x}_k^{t+1}, \mathbf{y}_k^{t+1}, \mathbf{z}_k^{t+1}; \mathbf{0}, \mathbf{g}_y^{t+1}, \mathbf{g}_z^{t+1})$	81	70	67	60

Note: each distance is calculated 504 times.

Table III reports the average annual growth of the LHM indicator under both for the [24] and the [25] specifications as well as under nonconvex and convex technologies. Looking at the average annual growth of the LHM indicator in Table III suggests that the use of the convex rather than a nonconvex technology renders higher productivity growth. This can perhaps partially be explained by the presence of infeasibilities as discussed above. The contribution of the economic and environmental productivity growth to the overall TFP growth (as explained in Equations 15-16) suggests the same pattern for both convex and nonconvex technologies. Specifically, the economic performance positively contributes to the overall TFP growth, whereas a negative contribution is observed for environmental performance. It is noteworthy that the magnitude of the environmental performance contribution varies little (e.g., -2.64% for nonconvex technology and -2.69% for convex technology under the [24] specification). Thus, the main differences in the TFP growth rate come from the economic component. Indirectly, this suggests that the environmental performance of the Chinese textile companies is rather similar, whereas the economic performance varies more substantially across the companies. This stresses the importance of the sector-wide environmental policies that could trigger further TFP growth.

TABLE III
AVERAGE GROWTH RATES OF TFP COMPONENTS

Model specification	Murty et al. [24]		Baležentis et al. [25]	
	Nonconvex	Convex	Nonconvex	Convex
	Equation	Equation	Equation	Equation
	14B	14A	15B	15A
$LHM^{t,t+1}$	5.23%	8.53%	1.62%	8.26%
$LHM_{eco}^{t,t+1}$	7.87%	11.22%	2.60%	8.66%
$LHM_{env}^{t,t+1}$	-2.64%	-2.69%	-0.97%	-0.40%
$TEC^{t,t+1}$	-1.94%	-3.87%	-2.25%	-3.71%
$TEC_{eco}^{t,t+1}$	-1.94%	-4.09%	-1.65%	-4.10%
$TEC_{env}^{t,t+1}$	-0.01%	0.22%	-0.61%	0.39%
$TP^{t,t+1}$	2.80%	7.66%	4.24%	8.58%
$TP_{eco}^{t,t+1}$	1.59%	8.54%	2.85%	9.05%
$TP_{env}^{t,t+1}$	1.21%	-0.87%	1.39%	-0.46%
$SEC^{t,t+1}$	4.37%	4.74%	-0.36%	3.39%
$SEC_{eco}^{t,t+1}$	8.22%	6.77%	1.39%	3.71%
$SEC_{env}^{t,t+1}$	-3.84%	-2.04%	-1.76%	-0.33%

Note: the growth rate is computed by OLS regression.

The decomposition of the TFP growth with respect to the sources of growth qualitatively does neither on average depend heavily on the technology assumed (i.e., convexity) nor on the model specification ([24] vs. [25]). Under either the assumption of convexity or nonconvexity, the technical efficiency change contributes negatively to the TFP growth (except for $TEC_{env}^{t,t+1}$ under convexity), whereas the technical progress and scale efficiency change contribute positively (except for $TP_{env}^{t,t+1}$ under convexity, $SEC^{t,t+1}$ under nonconvexity, and $SEC_{env}^{t,t+1}$ under both nonconvexity and convexity). These results suggest that there are some textile companies that depart from the efficiency frontier over time. This issue needs to be addressed by identifying those companies and the underlying causes of inefficiency.

The decomposition of the technical efficiency change component with regards to the contributions of the economic and environmental performance sheds more light on the sources of inefficiency. The economic performance appears to be the key source of the negative contribution of the technical efficiency change component. The environmental performance contribution is close to nil (especially for the [24] specification), which seems to suggest that there is quite a need for further improvements in the sense of the environmental performance of China's textile companies.

The technical progress is driven by both economic and environmental performance improvements. Under the nonconvex technology for the [24] specification, the contributions of the economic and environmental performance are of a similar magnitude, namely 1.59% and 1.21%, respectively. In the case of the nonconvex technology, the directions of the two components differ: the economic performance shows a positive contribution of 8.54% p.a., whereas the environmental performance is associated with a decline in the TFP of 0.87% p.a. In any case, the economic

performance seems to dominate the environmental component. Note that the very magnitude of the technical progress contribution to the TFP changes dramatically when switching from the convexity to the nonconvexity assumption. The analysis for the [25] specification is very similar.

Scale efficiency change is an important measure when firm-level micro-data are used for the analysis since it can guide policy makers in deciding whether concentration should be encouraged within a certain industry or not. The results in Table III suggest that there have been TFP gains due to increasing scale efficiency (except for $SEC^{t,t+1}$ under nonconvexity in the [25] specification). This indicates that China's textile firms have adjusted their scale of operation so as to approach the most optimal scale size (i.e., the constant returns to scale region on the production frontier).

The cumulative growth trends for the components of the LHM TFP indicator are presented in Fig. 1 for the [24] specification. The environmental performance is clearly poorer than the economic performance for this sample of the Chinese textile industry. The results are impressive anyway as the cumulative TFP growth due to economic performance stands at 105%, whereas the TFP loss due to environmental performance is 20% (for the convex technology). This still leaves substantial TFP gains of some 80% over 2001-2010. The contribution of the economic performance goes down to 69% in case when a nonconvex technology is assumed and the contribution of the environmental performance does not change much. A slightly negative trend is observed for the environmental performance with certain cyclical movements. The economic contribution tends to increase until about 2007. Thereafter, the economic recession has had an impact resulting in a subdued further growth.

For the [25] specification, the cumulative growth trends for the components of the LHM TFP indicator are presented in Fig. 2. While the absolute growth numbers are obviously different, the basic trends are very similar to the ones described for Fig. 1.

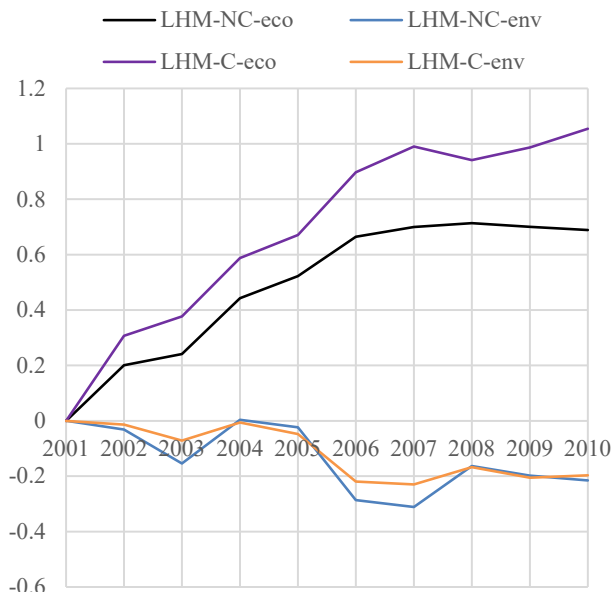


Fig. 1. Cumulative average LHM productivity indicator for the whole group of textile firms – Murty’s approach with PDFs

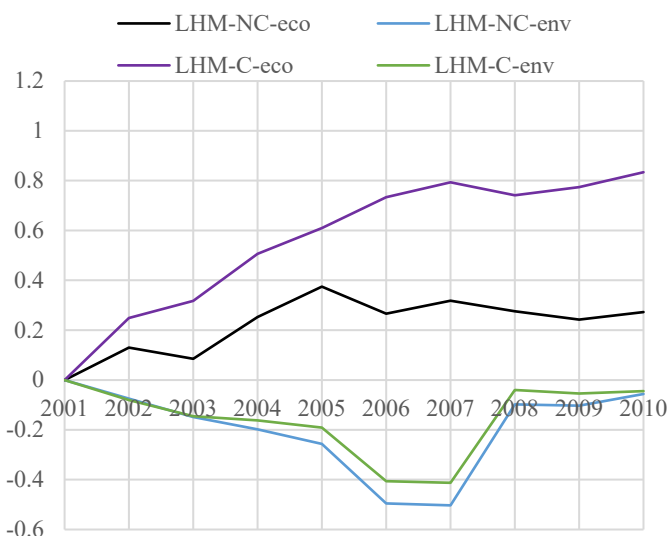


Fig. 2. Cumulative average LHM productivity indicator for the whole group of textile firms – Baležentis’s approach with PDFs

We employ a nonparametric test statistic proposed by [39] that basically compares two densities to formally assess their differences. This Li-test has been refined by [40] and [41], among others. This nonparametric test evaluates the differences between entire distributions rather than focusing on, e.g., the first moments only (as, for instance, the Wilcoxon signed-ranks test does). It tests the statistical significance of differences between two kernel density estimates, f and g , of a random variable x . The null hypothesis states that both density functions are almost everywhere equal ($H_0: f(x) = g(x)$ for all x). The alternative hypothesis negates this equality between both density functions ($H_1: f(x) \neq g(x)$ for some x). This nonparametric test statistic is valid for both dependent and independent variables: notice that dependency characterizes nonparametric frontier estimators (e.g., efficiency levels depend on sample size, among others).

TABLE IV
LI-TEST RESULTS OF EACH COMPONENT BETWEEN CONVEX AND NONCONVEX MEASURES

Technology	Measures	Components	Murty et al. [24]	Baležentis et al. [25]
Overall	Convex vs. Nonconvex	TFP	0.7332	-0.0406
		TEC	49.3790***	22.9629***
		TP	2.2344**	14.0882***
		SEC	4.5501***	8.2180***
Economic	Convex vs. Nonconvex	TFP	0.8933	-0.2395
		TEC	23.9865***	7.7995***
		TP	4.6679***	10.4596***
		SEC	1.2435	1.8795**
Environment	Convex vs. Nonconvex	TFP	2.0202**	5.6964***
		TEC	126.3036***	25.1841***
		TP	9.5031***	33.2239***
		SEC	13.3830***	2.9683***

Li-test critical values at 1% level=2.33(***) ; 5% level=1.64(**) ; 10% level=1.28(*).

First, we conduct Li-test statistics between convex and nonconvex measures for each component under overall

technology, economic technology and environmental technology, respectively. The results of this Li-test (T_n values) are shown in Table IV. From Table IV, the following

observations can be made. First, given the overall technology assumption, technical efficiency change (TEC), technology productivity (TP) and scale efficiency change (SEC) between convex and nonconvex measures all differ significantly at the 5% significance level. In contrast, total factor productivity (TFP) between convex and nonconvex measures has the same distribution. Second, considering the economic technology assumption, both TEC and TP between convex and nonconvex measures are significantly different at the 1% significance level. However, TFP are identically distributed. The SEC component is identically distributed for the [24] specification, while it is significantly different for the [25] specification. Third, considering the environmental technology assumption, TFP, TEC, TP, and SEC all differ significantly at the 5% significance level. Hence, we can notice that only under the environmental technology there are significant differences between all components of convex and nonconvex measures, especially the TFP component, which has the same distribution under both the overall and economic technology.

Furthermore, given a convex or a nonconvex technology, we

are interested in knowing whether the overall technology is distributed differently from the economic technology, or whether the overall technology is distributed differently from the environmental technology, or whether the economic technology is distributed differently from the environmental technology. Therefore, we again perform Li-test statistics and report the results of these Li-test (T_n -values) statistics in Table V.

From Table V, the following conclusions can be deduced. First, given a convex technology, it can be seen that only TEC has a significant difference between the overall technology and the economic technology at the 1% significance level, while TP and SEC all have the same distribution. TFP is identically distributed for the [24] specification, but it is significantly different for the [25] specification. Secondly, all four components (TFP, TEC, TP and SEC) differ at the 1% significance level between the overall and environmental technologies, as well as between the economic and environmental technologies.

TABLE V
LI-TEST RESULTS UNDER A TWO-BY-TWO TECHNOLOGY COMPARISON FOR A GIVEN CONVEX OR NONCONVEX MEASURE

Measure	Technology	Components	Murty et al. [24]	Baležentis et al. [25]
Convex	Overall vs. Economic	TFP	0.8846	3.8624***
		TEC	6.9214***	3.8104***
		TP	1.1703	0.9667
		SEC	-0.7620	-0.2634
	Overall vs. Environment	TFP	11.6806***	35.7535***
		TEC	42.6417***	11.8864***
		TP	10.5194***	8.8481***
		SEC	8.2165***	6.1541***
	Economic vs. Environment	TFP	6.07175***	17.4146***
		TEC	13.8073***	14.3060***
		TP	8.08641***	12.0176***
		SEC	7.8465***	7.1724***
Nonconvex	Overall vs. Economic	TFP	8.9014***	2.4956***
		TEC	-1.043	-0.6862
		TP	5.8316***	-0.1593
		SEC	17.7807***	6.3449***
	Overall vs. Environment	TFP	7.3340***	6.6872***
		TEC	94.9980***	1.8717**
		TP	4.1715***	2.3922***
		SEC	0.6082	5.1276***
	Economic vs. Environment	TFP	3.36382***	3.9526***
		TEC	80.9652***	1.7426**
		TP	0.2247	2.3922***
		SEC	8.6894***	-1.2943

Li-test critical values at 1% level=2.33 (***) ; 5% level=1.64 (**); 10% level=1.28 (*).

Second, given a nonconvex technology, it can be observed that TFP and SEC have significant differences between the overall technology and the economic technology at the 1% significance level. Only the distribution of TEC is the same. TP is significantly different for the (2012) specification, but identical for the (2021) specification. Furthermore, TFP, TEC and TP are significantly different between the overall

technology and the environmental technology at the 1% significance level. Here, only SEC is identically distributed for the [24] specification, while it is significantly different for the [25] specification. Finally, the TFP and TEC differ significantly between the economic technology and the environmental technology at the 1% significance level. Only TP (SEC) is identically distributed under the Murty et al. (Baležentis et al.) specification.

Up to now we have focused on the aggregate measures of TFP growth levels and dispersion. Still, the analysts may be interested in the performance of individual firms and the possibilities for improvement thereof. The present contribution focuses on the issue of nonconvexity in the LHM TFP analysis. Therefore, we compare the cumulative TFP growth for each firm following either the assumption of convexity or nonconvexity in Figs. 1 and 2. The resulting correlation coefficient is 0.77. This seems to suggest that the aggregate measures are likely to follow similar trends irrespectively of the maintained assumption. However, significant departures from equality of the convex and nonconvex TFP measures is already documented in Table IV.

But, these aggregate results may well hide substantial differences at the level of individual observations. Given our focus on the impact of convexity for engineering management, Table VI reports contradictory results for the LHM TFP indicator between convex and nonconvex technologies for both the [24] and the [25] specifications: we simply count the observations that have a contradictory sign of TFP. While contradictory results have, e.g., been reported by [17] for the Luenberger versus the LHM TFP indicator (see their Table 2), this is to the best of our knowledge the first time in the productivity literature that contradictory results are reported for some given productivity index or indicator for the nonconvex versus the convex technologies.

TABLE VI
CONTRADICTIONARY RESULTS OF TFP CHANGES (2001-2010)

Model	Contradictory results under nonconvex vs convex technologies	
	Murty et al. [24]	Baležentis et al. [25]
2001-2002	16/56	9/56
2002-2003	10/56	9/56
2003-2004	12/56	10/56
2004-2005	15/56	15/56
2005-2006	11/56	12/56
2006-2007	13/56	13/56
2007-2008	8/56	7/56
2008-2009	12/56	13/56
2009-2010	9/56	10/56

Note: If the sign of TFP change is inversed, it is a contradictory value.

Two observations emerge from analysing Table VI. First, contradictory results are present for each and every year of the sample. Second, contradictory results affect between 12.5 % (7/56) to 28.5% (16/56) of observations. Thus, the problem affects about a third of all observations. The inevitable conclusion is that engineering managers must opt for the correct specification of technology that is most suitable for the engineering reality: i.e., nonconvexity.

IV. CONCLUSIONS

In this contribution, we have discussed an environmental LHM TFP indicator and its decomposition under convex and nonconvex technologies. The proportional distance functions have been defined so that the input distance function seeks to minimize the use of inputs, whereas the output distance

function seeks to expand (resp. contract) the production of desirable (resp. undesirable) outputs. The change in TFP is then factorized with respect to technical progress, technical inefficiency change, and scale inefficiency change. Furthermore, we decompose the TFP growth and its components with regards to the contributions due to economic and environmental performance.

The empirical example considered in this study focuses on a small sample of the Chinese textile industry. The data for 2001-2010 have been used to define the environmental production technology and to gauge the LHM TFP measures. An overall positive trend in TFP growth is observed for this sector. Meanwhile, the environmental performance gains were not very satisfactory.

Furthermore, for the first time in the productivity literature contradictory results have been reported for the LHM TFP indicator for the nonconvex versus the convex technologies: up to a third of the sample experiences some basic contradiction. This should make engineers distrust the convexity axiom that they have often shared with the economic tradition, but that has little place in the engineering tradition.

The results suggest several further policy implications. First, the negative contribution of the technical efficiency change implies that there is a need for knowledge spill-over to ensure that there are no firms isolated from the modern practices or lacking managerial skills. Second, the scale efficiency component is positive indicating that the structural dynamics have been favorable in terms of resource allocation and productivity. The environmental dimension requires attention from all the companies since the negative contribution of this performance dimension is evident irrespectively of the convex or nonconvex technology analysed. At the firm-level, we have shown that qualitative differences may exist depending on the convexity assumption.

Further research could apply different methodologies to identify the major sources of inefficiency and productivity change in China's textile industry. In this contribution, we relied on the deterministic framework. Future research could introduce statistical precision in the nonparametric frontier measures. Also, stochastic approaches could be followed by estimating parametric production frontiers with undesirable outputs.

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