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## **Anisotropic representations of visual space modulate visual numerosity estimation**

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## Abstract

1  
2 Humans can estimate the number of visually displayed items without counting. This capacity of  
3 numerosity perception has often been attributed to a dedicated system to estimate numerosity, or  
4 alternatively to the exploitation of various stimulus features, such as density, convex hull, the size of  
5 items, and occupancy area. The distribution of the presented items is usually not varied with eccentricity  
6 in the visual field. However, our visual fields are highly asymmetric. To date, it is unclear how  
7 inhomogeneities of the visual field impact numerosity perception. Besides eccentricity, a pronounced  
8 asymmetry is the radial-tangential anisotropy. For example, in crowding, radially placed flankers  
9 interfere more strongly with target perception than tangentially placed flankers. Similarly, in  
10 redundancy masking, the number of perceived items in repeating patterns is reduced when the items  
11 are arranged radially but not when they are arranged tangentially. Here, we investigated whether  
12 numerosity perception is subject to the radial-tangential anisotropy of spatial vision to shed light on the  
13 underlying topology of numerosity perception. In Experiment 1, observers were presented with varying  
14 numbers of discs, predominantly arranged radially or tangentially, and asked to report their perceived  
15 number. In Experiment 2, observers were presented with the same displays as in Experiment 1, and  
16 were asked to encircle items that were perceived as a group. We found that numerosity estimation  
17 depended on the arrangement of discs, suggesting a radial-tangential anisotropy of numerosity  
18 perception. Grouping among discs did not seem to explain our results. We suggest that the topology of  
19 spatial vision modulates numerosity estimation and that asymmetries of visual space should be taken  
20 into account when investigating numerosity estimation.

21 *Keywords:* numerosity estimation, spatial vision, crowding, redundancy masking, radial-  
22 tangential anisotropy

23

## 24           **Introduction**

25           Humans can perform numerosity estimations without counting. When the number of items is  
26 small - usually up to 4 items - people apprehend the number of items rapidly and without errors (i.e.,  
27 subitizing (Atkinson et al., 1976; Kaufman et al., 1949)). However, estimating higher numbers of objects  
28 is usually imprecise compared with subitizing. Different mechanisms have been proposed to underlie  
29 numerosity estimation. A prominent account of numerosity perception suggests that it is accomplished  
30 by a dedicated system - the approximate number system (ANS, also known as the "number sense"). The  
31 ANS has been suggested to extract the numerosity independently from other physical properties of the  
32 stimulus (Barth et al., 2003; Burr et al., 2018; Dehaene, 1992, 2011; Dehaene et al., 1998; Feigenson et  
33 al., 2004; Gilmore et al., 2011; Halberda & Feigenson, 2008; Lipton & Spelke, 2003; Xu et al., 2005).

34           Other accounts suggest that numerosity perception is not performed by independent  
35 mechanisms dedicated to numerosity but by exploiting stimulus properties such as item density (Dakin  
36 et al., 2011; Durgin, 2008), occupancy area (Allik & Tuulmets, 1991), or by combining and weighting  
37 multiple visual cues (Gebuis & Reynvoet, 2012b, 2012c). Studies investigating the role of density in  
38 numerosity perception have shown diverging results. Burr and Ross (2008) demonstrated that  
39 numerosity, just like other primary visual properties, is subject to adaptation, and the effect was  
40 dependent on the number of items but not on other properties such as size or density. Hence, the  
41 authors suggested that numerosity is an independent visual property (see also Ross & Burr, 2010).  
42 Anobile et al. (2014) also suggested separate mechanisms for numerosity and density, supported by  
43 evidence that discrimination thresholds of high and low-density displays followed two distinct  
44 psychophysical functions (Weber's law and a square root function for low- and high-density displays,  
45 respectively). However, density and numerosity are physically indivisible, as density is calculated by  
46 dividing numerosity by the total area (Tibber et al., 2012). Dakin et al. (2011) showed that both  
47 numerosity and density judgments were biased by the size of the stimulus, which was interpreted to

48 imply that numerosity perception and density perception share a common metric (see also Tibber et al.,  
49 2012).

50           In addition to density, several other physical properties of displays have been shown to affect  
51 numerosity perception. For example, in the occupancy model, Allik and Tuulmets (1991) proposed that  
52 each presented item occupies a given circular region, and the total area collectively occupied by items  
53 (instead of the number of items per se) determined the perceived numerosity: When items are  
54 positioned too close to each other, the occupied regions overlap, resulting in lower perceived  
55 numerosity (see also, Allik & Raidvee, 2021). While proximity according to the occupancy model yields  
56 underestimation, varying proximity between subgroups of displayed items can yield more accurate  
57 performance. Specifically, when the presented items could be perceptually separated into subgroups,  
58 the number of items was enumerated more accurately and quickly ("groupitizing", Giovanni Anobile et  
59 al., 2020; Ciccione & Dehaene, 2020; Maldonado Moscoso et al., 2020; Pan et al., 2021). Hence, the  
60 spatial organization and perceptual grouping of items can modulate perceived numerosity. A similar  
61 effect of grouping has been shown with uniform versus regular patterns: Uniform patterns are often  
62 perceived to be more numerous than patterns that can be grouped into clusters (Frith & Frut, 1972;  
63 Ginsburg, 1976; Taves, 1941). Chakravarthi and Bertamini (2020) investigated numerosity estimation  
64 and crowded target discrimination using identical stimulus configurations, varying spacing and similarity  
65 among items that are both known to affect numerosity perception and crowding (see below). Based on  
66 their results that spacing and similarity impacted crowded discrimination and numerosity estimation  
67 differently, they suggested that underestimation in numerosity perception was not due to crowding but  
68 due to clustering among items, and that grouping may moderate both. Similarly, Im et al. (2016) found  
69 that the number of perceived groups predicted perceived numerosity, with smaller numerosity  
70 estimates when items were arranged in subgroups (yielding fewer perceived groups), suggesting that  
71 grouping between items plays a role in numerosity perception

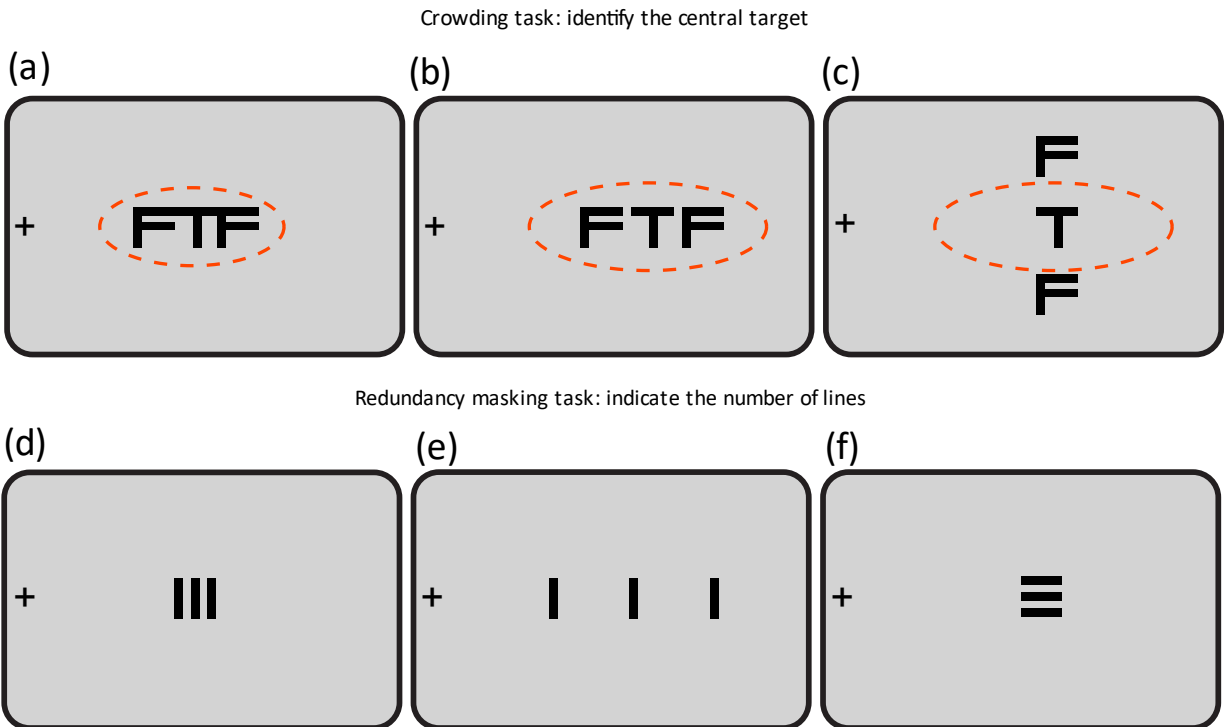
72 Another suggestion for factors modulating or determining numerosity estimates is that  
73 observers combine (and weight) information from various visual cues (including item size, aggregate  
74 surface, convex hull, and density) to estimate numerosity (Gebuis & Reynvoet, 2012a, 2012b, 2013,  
75 2014). What most experiments on numerosity perception have in common is that they usually apply  
76 stimulus features homogenously to the entire display, independent of stimulus locations in the visual  
77 field. However, our visual field has strong inhomogeneities (Abrams et al., 2012; Carrasco et al., 2001;  
78 Greenwood et al., 2017) which are likely to affect numerosity perception. One of the key factors that  
79 modulates perception is the eccentricity in the visual field. For example, a decrease in performance with  
80 increasing eccentricity has been shown for various tasks, including letter recognition (Gurnsey et al.,  
81 2011; Wolford & Hollingsworth, 1974; Zahabi & Arguin, 2014), conjunction search (Carrasco et al., 1995;  
82 Scialfa & Joffe, 1998), target detection (Meinecke & Donk, 2002), and vernier offset discrimination  
83 (Harris & Fahle, 1996; Levi & Waugh, 1994). How eccentricity modulates numerosity perception has also  
84 been investigated (Mengal & Matathia, 1980; Valsecchi et al., 2013). For example, it was found that the  
85 perceived number of items was lower when stimuli were presented in the periphery compared to  
86 central vision (Valsecchi et al., 2013). The authors suggested that the underestimation in the periphery  
87 could have been due to crowding where targets that are easily identified in isolation become difficult to  
88 discern when flanked by other items (Figure 1a, 1b; Bouma, 1970; Levi, 2008; Pelli & Tillman, 2008;  
89 Strasburger et al., 1991; Strasburger et al., 2011; Whitney & Levi, 2011). As crowding occurs when  
90 multiple objects interact, it is a plausible mechanism that could underlie underestimation in numerosity  
91 perception where multiple - often close-by - items are presented. Importantly, while crowding is usually  
92 assumed to affect target identification but not detection (Livne & Sagi, 2007; Pelli et al., 2004), recent  
93 studies showed that target parts were often unnoticed under crowding (Coates et al., 2017; Sayim &  
94 Taylor, 2019; Sayim & Wagemans, 2017). A particularly strong case of such 'omission errors' occurred  
95 when flankers and the target were the same. For example, when presenting three identical letters Ts in

96 the periphery, observers frequently reported only 2 letters (see also, Sayim & Taylor, 2019). This effect  
97 was termed "redundancy masking": The reduction of the number of perceived items in repeating  
98 patterns (Sayim & Taylor, 2019; Yildirim et al., 2020, 2021, 2022). Redundancy masking has been shown  
99 to occur when as few as 3 items were presented (Sayim & Taylor, 2019; Yildirim et al., 2020, 2021,  
100 2022). Notably, redundancy masking – as crowding - has a pronounced radial-tangential anisotropy: In  
101 crowding, radially placed flankers interfere more strongly with target perception than tangentially  
102 placed flankers (see Figure 1c, Greenwood et al., 2017; Kwon et al., 2014; Toet & Levi, 1992);  
103 redundancy masking is strong with radially arranged lines and absent with tangentially arranged lines  
104 (Figure 1d, e, f; Yildirim et al., 2020). As performance in most tasks deteriorates with increasing  
105 eccentricity (even if no contextual elements are presented), anisotropies such as the radial-tangential  
106 anisotropy are better suited to investigate to what extent numerosity perception is determined by  
107 similar contextual interactions as crowding and redundancy masking.

108         Here, we investigated whether numerosity perception is subject to a radial-tangential  
109 anisotropy to shed light on the underlying topology of numerosity perception. Specifically, we created  
110 displays that favored or did not favor these effects to occur (in 2 different alignment conditions:  
111 *tangential* and *radial*). We presented two types of arrangements of discs to produce weak or strong  
112 interference among the presented discs. To obtain a weak interference condition, close-by discs were  
113 predominantly arranged tangentially (*tangential* condition; Figure 2a); to obtain strong interference,  
114 they were predominantly arranged radially (*radial* condition; Figure 2b). In the *tangential* condition,  
115 elliptical zones around each disc that were expected to yield strong interference from neighboring discs  
116 within the zones ("crowding" zones) were "protected" by preventing discs from being positioned in  
117 these regions (hence, allowing tangential arrangements of discs, radial "protection zones" were used). In  
118 the *radial* condition, "protection zones" were perpendicular to these interference regions (i.e.,  
119 tangential oriented), allowing discs to fall into other discs' interference regions (Figure 2e). We varied

120 the size of the interference and protection zones as a function of eccentricity. Other physical properties  
121 (convex hull, occupancy area, density etc.) did not differ in the two conditions. In two experiments,  
122 participants viewed *tangential* and *radial* displays and were asked to perform the numerosity estimation  
123 task (Experiment 1) and the grouping task (Experiment 2). In Experiment 1, we tested whether the  
124 alignment condition (*radial* vs. *tangential*) influenced the perceived numerosity. Observers were asked  
125 to indicate the number of discs on each display. We found that the estimates of the number of discs  
126 were lower in the *radial* (strong interference) compared to the *tangential* (weak interference) condition.  
127 In Experiment 2, we tested whether there were any differences in the perceived number of groups in  
128 the two conditions, and thereby whether grouping could underlie the observed results in Experiment 1.  
129 For that aim, we asked participants to encircle the discs that they perceived to form groups.  
130 Interestingly, the results of Experiment 2 showed the opposite effect of the alignment condition on the  
131 perceived number of groups than the main experiment: The average number of groups reported by  
132 observers was larger in the *radial* compared to the *tangential* condition. This result suggests that the  
133 relatively lower estimates in the *radial* condition compared to the *tangential* condition (Experiment 1)  
134 was not likely caused by factors related to perceptual grouping as tested in Experiment 2. Overall, our  
135 results showed a pronounced radial-tangential anisotropy of numerosity perception, suggesting a similar  
136 underlying topology of spatial vision as in other types of contextual interactions.





137

138 **Figure 1** Illustration of crowding and redundancy masking. (a) When fixating the cross, identifying the  
 139 target "T" that is surrounded by 2 flankers "F", is usually difficult when flankers are positioned inside the  
 140 interference ("crowding") region (indicated by the dashed ellipse). (b) The interference region is  
 141 eccentricity-dependent: increasing target eccentricity increases the size of the interference region. (c)  
 142 The interference region is anisotropic: Flankers cease to interfere at smaller distances in tangential (c)  
 143 compared to radial (b) directions. (d) Redundancy masking is the reduction of the number of perceived  
 144 items in repeating patterns. When presenting 3 close-by aligned vertical lines in the periphery, most  
 145 observers reported only 2 lines. (e) Redundancy masking was weaker with large compared to small (d)  
 146 spacings (Yildirim et al., 2020). (f) There was no redundancy masking when lines were arranged  
 147 tangentially.

148

149 **Method**

150 **Experiment 1: Numerosity Estimation**

151 In Experiment 1, we tested whether the radial-tangential anisotropy of visual space impacted  
152 perceived numerosity.

153 ***Participants***

154 Twenty-one healthy participants (7 males, 14 females; mean age: 24.1 years, ranging from 19 to  
155 31) participated in the experiment. All participants were naïve as to the purpose of the study.  
156 Participants either received monetary compensation or participated without compensation. All  
157 participants reported normal or corrected-to-normal visual acuity and signed informed consent prior to  
158 the experiment. The experiment was approved by the ethics committee of the ULille SHS, University of  
159 Lille.

160 ***Apparatus***

161 The experiment was programmed in PsychoPy coder v3.1.2 (Peirce, 2009) and ran on a desktop  
162 PC. All stimuli were presented on a Vision Master Flat Square CRT monitor (Iiyama MS103DT), with a  
163 resolution of 1280 × 1024 pixels (refresh rate was set at 100 Hz). During the experiment, participants sat  
164 in front of the monitor with a chin rest at a distance of 57 cm from the monitor. All experiments were  
165 conducted in a dim experimental room.

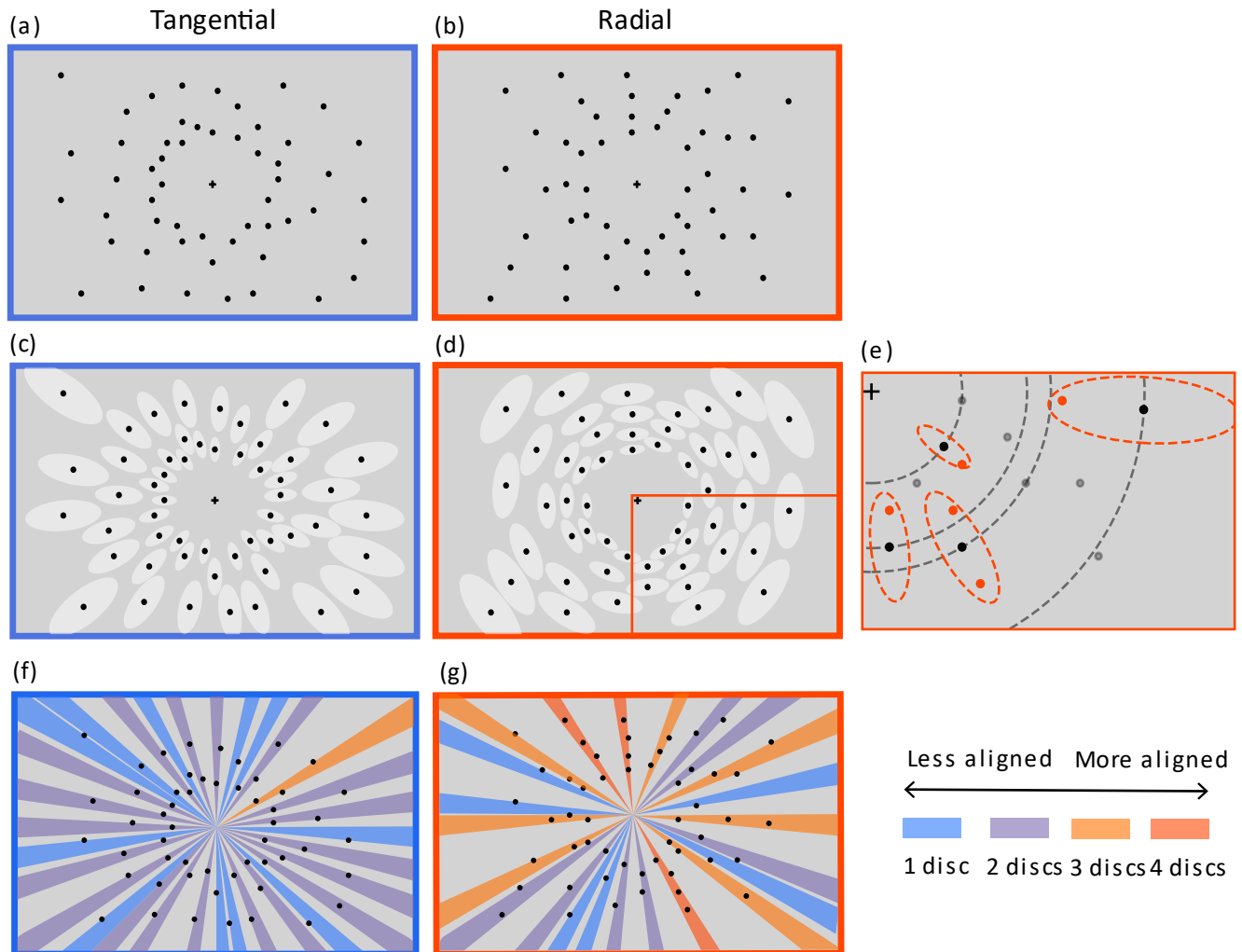
166 ***Stimuli***

167 Stimuli consisted of black discs ( $0.9 \text{ cd/m}^2$ ; radius:  $0.25^\circ$ ) presented on a gray background ( $25$   
168  $\text{cd/m}^2$ ). In five numerosity range conditions, discs were presented within rectangular regions of different  
169 sizes (width × height:  $19.5 \times 11.5$ ;  $21.5 \times 13.5$ ;  $25 \times 16.5$ ;  $27 \times 18.5$ ;  $30 \times 21$  degrees of visual angle that  
170 occupy 30%, 40%, 50%, 60%, and 70% of the screen, respectively), each corresponding to one of the 5  
171 different numerosity ranges (21 – 25; 31 – 35; 41 – 45; 49 – 53; 54 – 58). No discs were presented within  
172 a circular region (radius:  $3.8^\circ$ ) around fixation. There were two types of disc arrangements: tangential

173 and radial, illustrated in Figure 2. We surrounded each disc with a virtual "protection zone" free of any  
174 other disc. The size of the "protection zone" was based on common estimates of the size of the  
175 interference region in crowding (e.g., Bouma (1970), Toet & Levi (1992)). Both the major axis and the  
176 minor axis of the "protection zone" were determined by target eccentricity: the major axis was set to  
177  $0.25 \times$  eccentricity and the minor axis to  $0.1 \times$  eccentricity (corresponding to a minimum distance of 0.2  
178 and  $0.5 \times$  eccentricity when two discs were tangentially or radially aligned). To generate a *tangential*  
179 display (Figure 2a, c and f), a random position was chosen to place the first disc with its corresponding  
180 (radially extended) "protection zone." All the other discs were added with their "protection zones"  
181 iteratively on the displays with the constraint not to overlap with any of the "protection zones" of other  
182 discs, until no disc could be positioned onto the display without overlapping "protection zones." In the  
183 *radial* condition (Figure 2b, d, and g), displays were generated the same way as the *tangential* displays,  
184 except that the "protection zones" were rotated by  $90^\circ$  compared to the *tangential* condition.  
185 Therefore, in the *radial* condition, "protection zones" were orthogonal to the major axis of the  
186 interference region (Figure 2d, e). For each numerosity range, we generated 5000 displays for each  
187 condition (*tangential* and *radial*). We calculated convex hull, occupancy area, average spacing, average  
188 eccentricity, and density for each generated display and selected displays from the *tangential* and *radial*  
189 conditions that matched their physical properties (see Supplementary Table S1). The density was  
190 measured by dividing numerosity by occupancy area, excluding the central region where no discs were  
191 presented. As an insufficient number of displays in the smallest numerosity range could be matched, we  
192 generated an additional 5700 *radial* displays to obtain the required matches.

193

194



195

196 **Figure 2**197 Illustration of displays in the (a) *tangential* and (b) *radial* conditions. (c) and (d): **Illustration of the**198 **geometric principles** of the *tangential* and *radial* conditions. (c) In the *tangential* condition, each disc is

199 surrounded by a "protection zone" (indicated by the ellipses), allowing predominantly tangential

200 alignments of discs. No discs were positioned into any other disc's interference region zones. (d) Rotated

201 protection zones in the radial condition, favoring stronger interference. Here, a certain number of discs

202 was positioned inside other discs' interference regions. (e): Detail of the *radial* display, illustrating discs

203 (shown in red for illustration) in the interference region of other discs. (f) and (g) illustrate radial-

204 tangential alignment scores for the *tangential* and *radial* conditions, respectively.

205

206 ***Design and Procedure***207 At the start of each trial, a black fixation cross ( $0.75^\circ \times 0.75^\circ$ ) was presented at the center of the

208 screen. Observers initiated each trial by pressing the spacebar. The stimulus display was presented for

209 150 ms. Participants were instructed to respond by entering their best estimation of the number of  
210 presented discs with the numeric keypad. No feedback was provided. There was no time limit for  
211 participants to respond. Participants were not informed about the numerosity ranges prior to the  
212 experiment. Prior to each experimental block, participants viewed 5 reference displays in random order.  
213 The numerosities of the 5 reference displays were equally distributed around the averaged numerosity  
214 of the block ( $\pm 0.125$  and  $\pm 0.25$  times of the mean numerosity of the block). Each reference display was  
215 presented for 150 ms, and participants were informed about the actual numerosity of the display after  
216 the reference display offset.

217         There were two factors: Alignment condition (*tangential* vs. *radial*) and numerosity range (5  
218 levels: 21-25, 31-35, 41- 45, 49-53, and 54-58; for convenience, we use the first numerosity of each  
219 numerosity range to denote the actual numerosity range, i.e., N21, N31, N41, N49 and N54 denote  
220 numerosity range 21-25, 31-35, 41- 45, 49-53, and 54-58, respectively). Each participant performed 10  
221 blocks of 50 trials each. Within each block, each numerosity was presented 10 times (5 different  
222 displays, each repeated twice). Participants first completed each of the 5 numerosity ranges (in random  
223 order), followed by 5 blocks in the opposite order. The dependent variable was the deviation score (DV)  
224 of participants, calculated by subtracting the actual numerosity from participants' estimation for each  
225 trial. Hence, positive DVs represent overestimation; negative DVs represent underestimation. We also  
226 calculated the relative estimation error by dividing the DV by the numerosity of the display.

### 227 ***Data Analysis***

228         We conducted a within-subject ANOVA on DV scores with alignment condition and numerosity  
229 range as within-subject factors. We expected lower DVs in the radial compared to the tangential  
230 condition. The ANOVA and pairwise analysis were performed with an open-source Python package,  
231 Pingouin version 0.5.1 (Vallat, 2018). Estimates outside of 3 standard deviations around the mean were

232 discarded independently for each numerosity range (0.4% of all trials). The same analyses were  
233 conducted on relative estimation error.

234 **Radial alignment scores (RAs).** We calculated RAs as measures of how well discs were radially  
235 aligned in a display. RAs were calculated individually for each display by rotating a circle sector with an  
236 angle of  $6^\circ$  (half the angle of the minor axis of the protection zones) around fixation for a complete  
237 rotation and counting the number of discs falling in the sector at each location a new disc fell into the  
238 trailing edge of the sector (i.e., when the edge of the circle sector aligned with a disc center; Figures 2f  
239 and 2g). Neighboring circle sectors ("alignment regions") did not overlap. The procedure was repeated  
240 with each disc in the display as starting disc, always performing a complete rotation. For each rotation,  
241 the proportion of the circle sectors that contained 3 (the minimum number of items to obtain  
242 redundancy masking) or more discs was calculated. For example, if there were 20 circle sectors in one  
243 rotation and 10 of them contained 3 (or more) discs, the proportion would be 0.5. The RA of that display  
244 was the averaged proportion across all rotations for that display.

245 **Crowding strength.** The number of discs that was positioned in other discs' interference regions  
246 varied in the *radial* condition but not in the *tangential* condition since no discs could be positioned into  
247 the interference region of other discs (Figure 2c; by definition, what we denote as the "crowding  
248 strength" was 0 in all tangential displays). To quantify "crowding strength" in the *radial* condition, we  
249 calculated the number of discs per display that were positioned in other discs' interference regions. The  
250 average crowding strength was  $1.3 \pm 1.1$ ,  $2.6 \pm 1.3$ ,  $4.8 \pm 2.4$ ,  $6.6 \pm 2.9$ , and  $7.1 \pm 3.1$  for N21, N31, N41,  
251 N49, and N54, respectively.

252 **Partial correlations.** We calculated partial correlations between (1) RAs and DVs and (2)  
253 crowding strength and DVs, controlling for numerosity. To ensure that RAs, crowding strength, and DVs  
254 were comparable across numerosity ranges, they were normalized in the linear regression to predict  
255 numerosity.

## 256 **Experiment 2: Grouping into clusters**

257           In Experiment 2, we tested whether the number of perceived groups in the *radial* and *tangential*  
258 conditions differed. If the number of perceived groups was lower in the *radial* than in the *tangential*  
259 condition, grouping among discs could be a factor contributing to the effect found in Experiment 1. If  
260 the number of perceived groups was similar in the *radial* and the *tangential* displays, the results would  
261 suggest that grouping is an unlikely factor underlying the effect observed in Experiment 1.

### 262 **Participants**

263           Thirty healthy participants (4 males, 26 females; mean age: 19.7 years, ranging from 18 to 24)  
264 participated in Experiment 2. All participants were students at the University of Lille or the KU Leuven,  
265 and naïve as to the purpose of the study. All participants received course credits for their participation.  
266 All participants reported normal or corrected-to-normal visual acuity and signed informed consent prior  
267 to the experiment.

### 268 **Apparatus**

269           The experiment was programmed in PsychoPy coder v2.1.0 (Peirce, 2009) and ran on a desktop  
270 PC. All stimuli were presented on an LCD display with a resolution of 1960 × 1080 pixels. During the  
271 experiment, participants sat in front of the monitor with a chin rest at a distance of 57 cm from the  
272 monitor.

### 273 **Stimuli**

274           The stimuli were identical to the stimuli in Experiment 1.

### 275 **Design and Procedure**

276           The design and procedure were identical to Experiment 1 except for the following changes:  
277 Participants were asked to encircle the discs that they perceived as a group, using the mouse (as a  
278 'pen'). Each display was presented until participants had finished the trial (unlimited viewing time).  
279 Participants were presented with the same displays that were used in Experiment 1. Each participant

280 was presented with one-third of the total number of displays (250 displays) of Experiment 1 to limit the  
 281 duration of the experiment (about 100 minutes per participant). There were 30 participants (hence, 10  
 282 responses per display).

### 283 **Data Analysis**

284 The analyses were identical to the ANOVA analysis in Experiment 1, except that the dependent  
 285 variable was the number of perceived groups. The number of groups that participants encircled for each  
 286 display corresponded to the number of perceived groups in the analysis.

287

## 288 **Results**

### 289 **Experiment 1: Numerosity Estimation**

290 Figure 3a shows the average deviation scores (DVs) for the *tangential* and the *radial* condition  
 291 separately for each numerosity range. A repeated measures ANOVA with alignment condition  
 292 (*tangential* and *radial*) and numerosity range (N21, N31, N41, N49, and N54) as factors showed a main  
 293 effect of alignment condition ( $F(1, 20) = 13.45, p < .005, \eta_p^2 = .40$ ) on DVs. Participants reported fewer  
 294 discs in the *radial* ( $DV = 1.64 \pm 8.65$ ) compared to the *tangential* condition ( $DV = 2.66 \pm 8.78$ ). Pairwise  
 295 comparisons with Hochberg FDR correction showed significant differences between the *tangential* and  
 296 the *radial* conditions in all numerosity ranges (N31:  $t(20) = 2.66, p < .05$ , Cohen's  $d = 0.12$ ; N41:  $t(20) =$   
 297  $2.32, p < .05$ , Cohen's  $d = 0.10$ ; N49:  $t(20) = 3.43, p < .005$ , Cohen's  $d = 0.15$ ; N54:  $t(20) = 3.55, p = .005$ ,  
 298 Cohen's  $d = 0.16$ ), except for the smallest one (N21: ( $t(20) = 0.85, p = .40$ , Cohen's  $d = 0.04$ ). We also  
 299 found a main effect of numerosity range with lower DVs for small numerosities. ( $F(4, 80) = 3.96, p < .05$ ,  
 300  $\eta_p^2 = .17$ ). A significant interaction between alignment condition and numerosity range ( $F(4, 80) = 2.68, p$   
 301  $< .05, \eta_p^2 = .12$ ) indicated that the difference between the *tangential* and the *radial* conditions  
 302 increased with larger numerosities. Figure 3b shows the average relative estimation error for each  
 303 condition. We also conducted a repeated measures ANOVA on average relative estimation error with



304 alignment condition and numerosity range as within-subject factors. We observed a main effect of  
305 alignment condition ( $F(1, 20) = 8.79, p < .01, \eta_p^2 = .31$ ) on relative estimation errors. No other significant  
306 effect was observed ( $ps > .05$ ).

307 To test whether radial alignment predicted DVs, we correlated radial-alignment scores (RAs) and  
308 DVs while controlling for numerosity (partial correlation, Figure 3b). For all numerosity ranges  
309 combined, the partial correlation was  $r = -0.40$  ( $p < .0001, CI\ 95\% [-0.50 -0.29]$ ), showing higher  
310 deviations scores with increasing RAs. Except for N21, the partial correlation between DVs and RAs  
311 showed a clear negative correlation when controlling for the effect of numerosity. These results showed  
312 that estimates were smaller when discs were more strongly radially aligned, at least for larger  
313 numerosities (N31 and above). The averaged RAs for separate numerosity ranges for both *tangential*  
314 and *radial* displays are shown in Supplementary Table S2. The partial correlations for the separate  
315 numerosity ranges are shown in Supplementary Table S3.

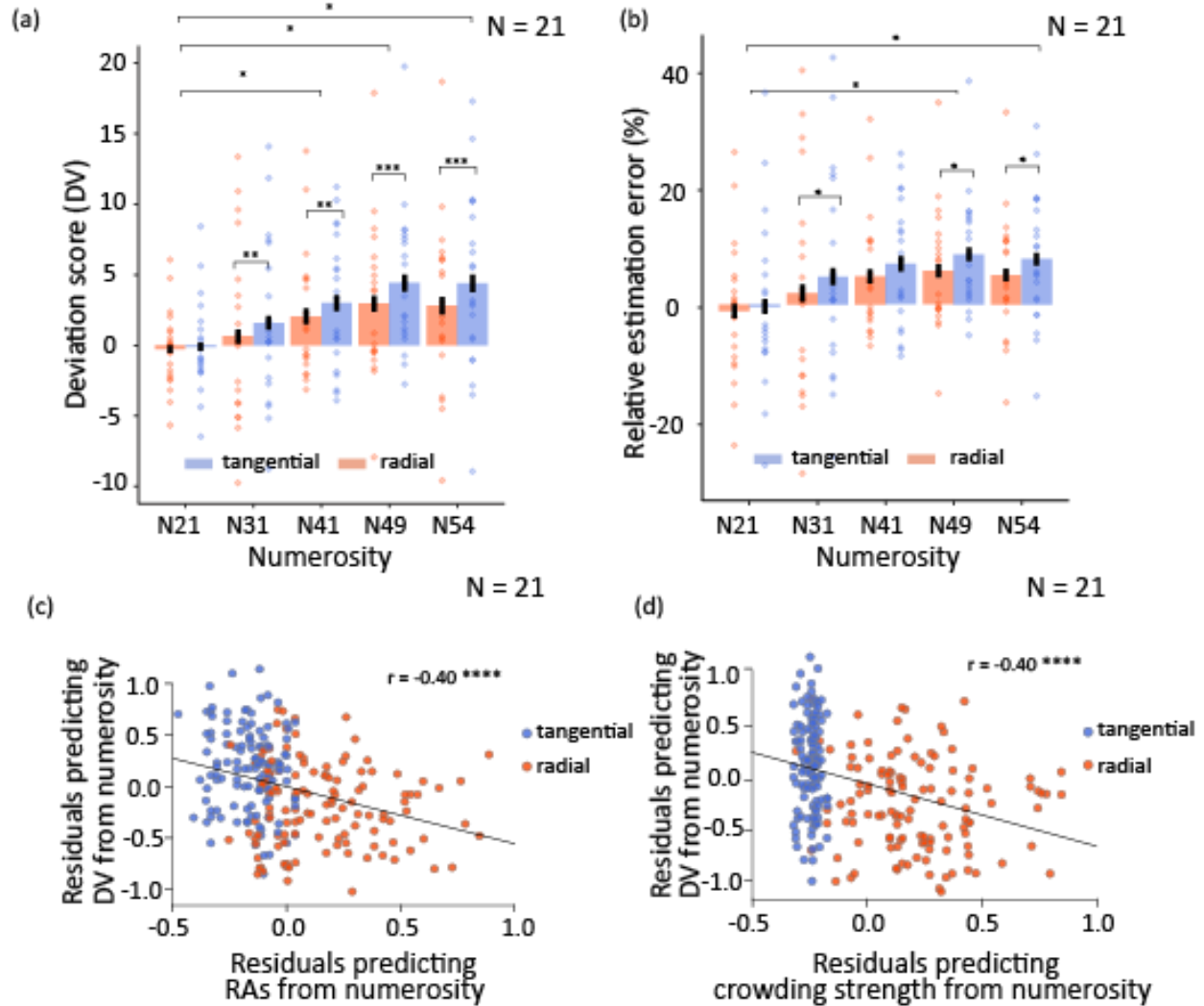
316 To test whether "crowding strength" predicted DVs, we correlated crowding strength and DVs  
317 while controlling for numerosity (partial correlations, Figure 3c). Results showed that the overall partial  
318 correlation coefficient was  $r = -0.40$  ( $p < .0001, CI\ 95\% [-0.50 -0.29]$ ). Hence, there was a clear negative  
319 correlation between the number of discs falling into the interference zone of other discs and numerosity  
320 judgments: The more discs were presented in other discs' interference zones, the lower the numerosity  
321 judgments. Supplementary Table S3 shows the partial correlations analysis of each numerosity range  
322 separately.

### 323 **Experiment 2: Grouping into clusters**

324 Figure 4 illustrates the task and response format in the grouping task for tangential and radial displays,  
325 respectively. A repeated measures ANOVA with alignment condition and numerosity range as factors  
326 showed a main effect of alignment condition ( $F(1, 9) = 6.91, p < .005, \eta_p^2 = .43$ ) on the perceived number  
327 of groups. Participants reported more groups in the *radial* ( $13.0 \pm 4.25$ ) compared to the *tangential*

328 condition ( $11.4 \pm 3.78$ ). Pairwise comparisons with Hochberg FDR correction showed significant  
329 differences between the *tangential* and the *radial* conditions in N21 ( $t(9) = 4.11, p < .01$ , Cohen's  $d =$   
330  $1.10$ ), but not in the other numerosity ranges (N31:  $t(9) = 2.08, p = .09$ , Cohen's  $d = 0.70$ ; N41:  $t(9) =$   
331  $2.08, p = .09$ , Cohen's  $d = 0.67$ ; N49:  $t(9) = 1.58, p = .15$ , Cohen's  $d = 0.40$ , N54:  $t(9) = 2.07, p = .09$ ,  
332 Cohen's  $d = 0.58$ ). Unsurprisingly, there was also a main effect of numerosity range on the perceived  
333 number of groups ( $F(4, 36) = 101.94, p < .001, \eta_p^2 = .92$ ), showing that more groups were perceived with  
334 larger numerosities. No interaction between alignment condition and numerosity range was observed  
335 ( $F(4, 36) = 0.58, p = .68, \eta_p^2 = .06$ ). Supplementary Table S4 summarizes the average perceived number  
336 of groups for each numerosity range in the *tangential* and the *radial* condition. Importantly, the two  
337 alignment conditions affected numerosity estimations (Experiment 1) and the perceived number of  
338 groups (Experiment 2) differently: numerosity estimation was lower and the perceived number of  
339 groups higher in the *radial* compared to the *tangential* condition.

340



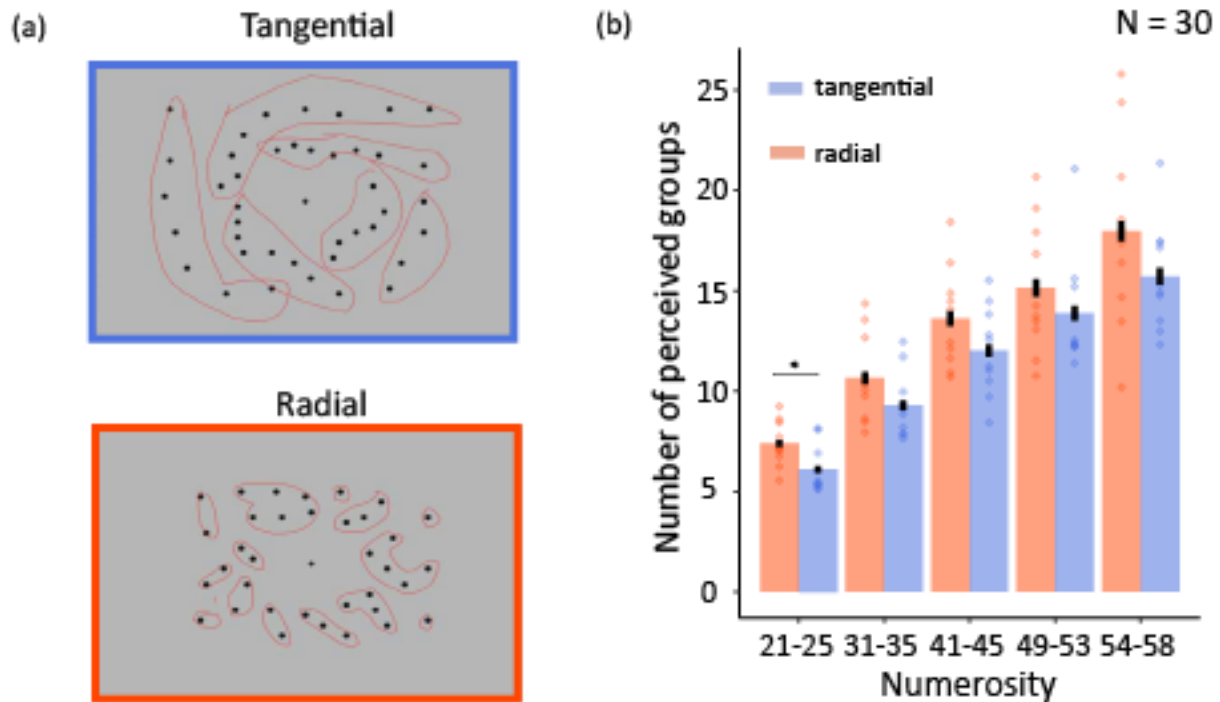
341  
 342 **Figure 3.** Results of Experiment 1. (a): Deviation score (DV) as a function of numerosity. DVs of 0  
 343 represent no deviation from correct responses, negative DVs represent underestimations, and positive  
 344 DVs represent overestimations. Error bars indicate (+/-1) standard errors of the mean. Significant  
 345 pairwise comparisons are indicated with asterisks. Each data point shows the average scores for one  
 346 observer. (b): Relative estimation error as a function of numerosity. Error bars indicate (+/-1) standard  
 347 errors of the mean. Each data point represents the average percent changes of one observer. (c): Partial  
 348 correlation between DVs and radial alignment scores (RAs). when controlling for the effect of  
 349 numerosity. (d): Partial correlation between DVs and crowding strength when controlling for the effect  
 350 of numerosity. (\* $p < .05$ . \*\* $p < .005$ . \*\*\* $p < .001$ . \*\*\*\* $p < .0001$ .)

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356 **Figure 4.** Results of Experiment 2. (a) Illustration of Experiment 2 with possible responses for tangential  
 357 and radial displays. Each closed red shape was counted as one group of items. (b) The number of  
 358 perceived groups as a function of numerosity separated for the radial and tangential conditions. Error  
 359 bars indicate (+/-) standard errors of the mean. Each data point shows the average scores for one  
 360 observer. (\* $p < .05$ ).

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### Discussion

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We investigated to what extent the topology of spatial vision determined numerosity estimation. In particular, based on the radial-tangential anisotropy of spatial interactions in the peripheral visual field, we sought to investigate if numerosity estimation was subject to a similar radial-tangential anisotropy as crowding and redundancy masking. For that aim, we created displays in which neighboring items were predominantly arranged in either tangential or radial directions while keeping other features of the two types of displays, such as inter-item spacing, average eccentricity, convex hull,

369 and density as similar as possible. In Experiment 1, we asked participants to report the number of discs  
370 they perceived. We found that numerosity estimates were lower in the *radial* compared to the  
371 *tangential* condition. The analysis of radial alignment scores (RAs) showed that higher RAs yielded lower  
372 numerosity estimates. In the *radial* condition, the number of items falling into the interference regions  
373 of other items was taken as a measure of "crowding strength." We found that crowding strength  
374 predicted deviation scores (DVs): high crowding strength was associated with smaller numerosity  
375 estimates and vice versa. Grouping among items is a good predictor of crowding strength (Livne & Sagi,  
376 2007; Manassi et al., 2012; Sayim et al., 2010; Sayim et al., 2011; but see Melnik et al., 2018; Rummens  
377 & Sayim, 2019a). Grouping has also been shown to modulate numerosity perception (Chakravarthi &  
378 Bertamini, 2020; Ciccione & Dehaene, 2020; Im et al., 2016; Pan et al., 2021). To test whether the  
379 number of perceived groups was related to the relative underestimation in the *radial* compared to the  
380 *tangential* condition, we asked observers in Experiment 2 to encircle the discs they perceived as a group.  
381 We used the same displays in the grouping task as in Experiment 1. The results showed that the number  
382 of perceived groups in the *radial* condition was higher than in the *tangential* condition, i.e., the opposite  
383 pattern of results compared to Experiment 1: lower estimations (Experiment 1) and higher number of  
384 groups (Experiment 2) in the *radial* compared to the *tangential* condition. Hence, the perceived number  
385 of groups and the perceived numerosity were affected by alignment conditions differently. These results  
386 indicate that grouping is unlikely the cause for the different numerosity estimates in the *radial* and the  
387 *tangential* condition.

388 Crowding strongly limits peripheral vision (Bouma, 1970; He et al., 1996; Levi et al., 2002; Pelli et  
389 al., 2004), and was proposed to play a role in numerosity estimates (Anobile et al., 2015; Valsecchi et al.,  
390 2013). In particular, the relative underestimation of numerosities in dot displays presented in the fovea  
391 compared to the periphery suggested that mechanisms related to crowding might be an important  
392 factor in numerosity perception (Valsecchi et al., 2013). A potential role of crowding was also shown

393 when varying eccentricity: Numerosity estimates varied with eccentricity similar to crowding, with  
394 stronger interference (lower estimates) farther in the periphery (Valsecchi et al., 2013). However,  
395 performance in most tasks deteriorates with increasing eccentricity. For example, besides crowding  
396 (Levi, 2008; Pelli et al., 2004; Strasburger, 2020; Toet & Levi, 1992), performance in other tasks,  
397 including letter recognition (Gurnsey et al., 2011; Wolford & Hollingsworth, 1974; Zahabi & Arguin,  
398 2014), conjunction search (Carrasco et al., 1995; Scialfa & Joffe, 1998), target detection (Gruber et al.,  
399 2014; Meinecke & Donk, 2002), visual search (Carrasco & Frieder, 1997; Carrasco et al., 1998) and  
400 vernier offset discrimination (Harris & Fahle, 1996; Levi & Waugh, 1994) deteriorates with increasing  
401 eccentricity. Hence, eccentricity dependence is not sufficient to conclude that crowding-like  
402 mechanisms underlie numerosity estimation. In a recent study, crowding and numerosity perception  
403 were directly compared using identical stimulus configurations (Chakravarthi & Bertamini, 2020). Inter-  
404 item spacing and item similarity (same or opposite contrast polarity), both known to modulate crowding  
405 as well as numerosity estimates were varied. The results showed that spacing and similarity affected  
406 numerosity perception (in a 2AFC numerosity comparison task) and crowding (in an identification task)  
407 differently, suggesting a dissociation between numerosity perception and crowding. However, the  
408 different tasks and different task-relevancy of the presented items – a single relevant target or many  
409 relevant targets – render definite conclusions about the dissociation of crowding and numerosity  
410 perception difficult. For example, whether items are task-relevant or not has recently been shown to  
411 strongly modulate crowding, inverting the similarity rule of crowding (Rummens & Sayim, 2019b): When  
412 all items were task-relevant, performance was superior with target and flankers of the same compared  
413 to opposite contrast polarity. Similarly, small spacing between target and flankers does not always yield  
414 stronger crowding: Emergent features between the target and a flanker improved performance at small  
415 compared to larger distances in a crowding task (Melnik et al., 2020).

416           Importantly, crowding is usually assumed to impair target identification but not target detection  
417 (Andriessen & Bouma, 1976; Levi, 2008; Pelli et al., 2004; but see Allard & Cavanagh, 2011; Sayim &  
418 Wagemans, 2017). As underestimation in numerosity perception implies failures of detection, not  
419 discrimination, it might be suggested that crowding is an unlikely candidate to play a role in numerosity  
420 perception in general. However, recently it was shown that parts of the targets are often lost in  
421 crowding (Sayim & Wagemans, 2017). Such "omission errors" may well be due to the recently  
422 discovered phenomenon of redundancy masking, the reduction of the number of perceived items in  
423 repeating patterns (Sayim & Taylor, 2019; Yildirim et al., 2020, 2021). Although related to crowding, a  
424 key difference is that redundancy masking, unlike crowding, impairs the perception of the number of  
425 items (not their identity). As in numerosity estimation, a typical task to investigate redundancy masking  
426 is to ask participants to report the number of perceived items (however, see Sayim & Taylor, 2019, for a  
427 free verbal report and drawing task). Hence, there are obvious parallels between redundancy masking  
428 and numerosity perception, and redundancy masking could underlie underestimation in numerosity  
429 perception. Importantly, redundancy masking occurs for as few as three presented items, i.e., in the  
430 subitizing range (Yildirim et al., 2020) where reports are usually accurate (Atkinson et al., 1976; Jensen  
431 et al., 1950; Kaufman et al., 1949). Although clearly present for larger numbers of items, redundancy  
432 masking does not scale linearly with the number of items. For example, with three presented items of  
433 which only two are reported, one-third of all items are lost due to redundancy masking. While the  
434 absolute number of items lost due to redundancy masking increases with the number of presented  
435 items, the ratio decreases (Yildirim et al., 2020). Hence, the exact relation between redundancy masking  
436 and numerosity estimation still needs to be investigated, with future studies closing the gap between  
437 the paradigms typically used in numerosity perception and in redundancy masking, and shedding light  
438 on the extent of their similarities. Importantly, redundancy masking – as crowding – has a pronounced  
439 radial-tangential anisotropy: When peripherally presented lines were arranged radially, redundancy

440 masking was strong; when they were arranged tangentially, there was no redundancy masking (Yildirim  
441 et al., 2020). Here, we used this radial-tangential anisotropy to manipulate displays where discs were  
442 predominantly arranged tangentially or radially to test if radial arrangements would yield lower  
443 estimates than tangential arrangements. As expected, radial arrangements yielded lower estimates than  
444 tangential arrangements. Taken together, contextual interactions subject to radial-tangential  
445 anisotropy, and in particular redundancy masking, are promising phenomena that share characteristics  
446 with numerosity perception beyond eccentricity dependence.

447 Many physical characteristics of displays used in experiments on numerosity perception are potentially  
448 confounded with numerosity per se (Gebuis & Reynvoet, 2012c). Importantly, in our *tangential* and  
449 *radial* arrangements, we kept physical properties of the displays that have been shown to play a role in  
450 numerosity estimation as similar as possible, matching them in regard to items size (Allik et al., 1991;  
451 Ginsburg & Nicholls, 1988), occupancy area (Allik & Tuulmets, 1991), convex hull (Gilmore et al., 2016;  
452 Katzin, 2018), regularity (Franconeri et al., 2009; Ginsburg, 1976; Liu et al., 2018; Zhao & Yu, 2016),  
453 spatial clustering (Bertamini et al., 2018; Bertamini et al., 2016; Chakravarthi & Bertamini, 2020;  
454 Koesling et al., 2004), and texture density (Dakin et al., 2011). Controlling for these possibly confounding  
455 physical properties in the two conditions minimized the probability of factors related to these properties  
456 to account for the effect of our manipulation. Given the predominantly *tangential* or *radial*  
457 arrangements in the two conditions, some systematic structural differences are unavoidable. In  
458 particular, the discs in the *tangential* displays tend to be arranged into concentric patterns around  
459 fixation and in the *radial* displays into ray patterns. Importantly, while these structural differences  
460 between the displays may be a variable that modulates numerosity estimates, the findings in  
461 redundancy masking show strong differences between *tangential* and *radial* arrangements without any  
462 global, structural differences between *tangential* and *radial* arrangements. Moreover, redundancy  
463 masking has been shown to increase – not decrease – with diffused compared to focused attention



464 (Yildirim et al., in preparation). As focused spatial attention is considered not required in numerosity  
465 estimation (at least with relatively sparse displays; Anobile et al., 2020; Burr et al., 2010), redundancy  
466 masking would not be expected to cease in displays with larger numerosities.

467         While the number of discs, average eccentricity, average spacing, convex hull, and density were  
468 matched in the *tangential and radial* conditions, all displays contained density gradients with higher  
469 density in more central regions and decreasing density with increasing eccentricity. Hence, differences  
470 of the spatial distributions of the discs as a function of eccentricity in the two conditions were possible.  
471 For example, relatively more discs could be close to the center in one display, forming a higher local  
472 density region, compared to fewer discs close to the center in another display (with the same number of  
473 discs). The local density as a function of eccentricity (Supplementary Figure S1) captures such variations  
474 of display density. Differences in local densities could be a factor influencing numerosity estimates, for  
475 example, by yielding higher numerosity estimates for displays with high local densities compared to  
476 displays with low local densities. Such an effect would be expected if central regions were weighted  
477 more strongly than peripheral regions (Cheyette & Piantadosi, 2019; see also, Dandan et al., in  
478 preparation). A small subset of displays in the *tangential* condition had relatively high local densities  
479 compared to the average (Figure S1). However, the majority of these displays were not judged as more  
480 numerous than displays with lower local density, suggesting that local density differences between the  
481 *tangential* ('concentric') and the *radial* ('ray') conditions did not underlie differences of numerosity  
482 estimates. Note that relatively low density (due to relatively larger item size or smaller convex hull) has  
483 also been reported to yield higher numerosity estimates compared to displays with relatively high  
484 densities (Gebuis & Reynvoet, 2012c), however, in relatively uniform displays, without any systematic  
485 density variation with eccentricity as in our displays. If the structural differences per se irrespective of  
486 other variables (e.g., local density, overall density, convex hull, etc.) modulated numerosity perception,  
487 with generally lower estimates in ray compared to concentric patterns, radial-tangential anisotropies

488 may well underlie such a difference. Systematic investigations to explore if – and how – such structural  
489 differences and local density differences modulate numerosity estimations will shed light on their role in  
490 numerosity perception.

491         Our results showed that the relative underestimation in the *radial* compared to the *tangential*  
492 condition was primarily driven by larger numerosities, with significant differences observed in N31 to  
493 N54 but not for N21. Consistently, in the partial correlation analysis, we found that both RAs and  
494 crowding strength negatively correlated with estimations with large numerosities but not small  
495 numerosities (see Supplementary Table S3). The pronounced effect on large but not small numerosity  
496 ranges is not surprising as the radial-tangential manipulation of displays did not yield strong differences  
497 in the smallest numerosity (N21, see RAs, Supplementary Table S2). While density did not differ  
498 between the *radial* and *tangential* conditions within each numerosity range, densities did vary between  
499 numerosity ranges: Relative higher density in N21 compared to the other numerosity (see  
500 Supplementary Table S1). Anobile et al. (2014) suggested that numerosity discrimination and judgments  
501 based on density depend on the density of the displayed items, with numerosity discrimination  
502 occurring when display densities are less than 0.25 items/deg<sup>2</sup> and judgments based on density with  
503 larger densities of the displays. In our displays, the densities in the large numerosity ranges (N41, N49  
504 and N54) where we found differences between the *radial* and *tangential* displays fell into the  
505 'numerosity judgment' range suggested by Anobile et al. (2014). Hence, it is unlikely that judgments in  
506 these conditions were based on density (but see Dakin et al., 2011; Durgin, 2008).

507         In contrast to smaller numerosities (N21) where the number of discs was rather accurately  
508 estimated, it was overestimated with larger numerosities (N31 and more). The overestimation with  
509 larger numerosities diverged from the general underestimation found in most numerosity studies  
510 (Anobile et al., 2020; Au & Watanabe, 2013; Chakravarthi & Bertamini, 2020; Krueger, 1982, 1984; Liu et  
511 al., 2017; Liu et al., 2018). The direct estimation task, in contrast to the typical discrimination task, could

512 be one reason for the overestimation in our study. Similar overestimations were found when presenting  
513 regular and irregular dots array (28 – 46 dots), asking observers to estimate the number of dots (Alam et  
514 al., 1986). Also, when asking participants to report the number of items, Gebuis and Reynvoet (2012c)  
515 found that half of the participants overestimated and the other half underestimated the numerosities.  
516 We can exclude that the overestimation was due to the overall distribution of numerosities in different  
517 blocks as the same pattern of results also occurred in the first block that observers completed.  
518 Importantly, irrespective of the overall overestimation, which suggests a general bias, it is the relative  
519 underestimation in the *radial* compared to the *tangential* condition that shows the key estimation  
520 difference between the two conditions.

521         Perceptual grouping has been shown to modulate perceived numerosity (Chakravarthi &  
522 Bertamini, 2020; Im et al., 2016; Mazza & Caramazza, 2012). When items were arranged into clusters  
523 (Chakravarthi & Bertamini, 2020; Frith & Frut, 1972), perceived to contain a larger number of groups (Im  
524 et al., 2016), were grouped by connectedness (Franconeri et al., 2009) or by similarity grouping  
525 (connectedness, shape, proximity, and common region (Yu et al., 2019), observers tended to  
526 underestimate the numerosity compared to similar displays with weaker grouping. Hence, grouping  
527 among items may have modulated the perceived numerosity in the present study as well. For example,  
528 the relative underestimation in the *radial* compared to the *tangential* condition could have been driven  
529 by more grouping (and therefore fewer groups) in the *radial* compared to the *tangential* displays. In  
530 Experiment 2, we investigated how the discs in our displays were perceived to groups and whether  
531 grouping differences between the conditions could underlie the pattern of results in Experiment 1.  
532 Interestingly, the average number of perceived groups was higher in the *radial* than in the *tangential*  
533 condition, in contrast to number estimates which were lower in the *radial* compared to the *tangential*  
534 condition. Hence, this result shows that displays with low (high) numbers of perceived groups did not  
535 yield low (high) numerosity estimates. These results suggest that the relative underestimation in the

536 *radial* compared to the *tangential* displays was not due to a smaller number of groups in the *radial*  
537 compared to the *tangential* condition: Grouping into clusters seems unlikely to play an important role in  
538 our results. However, while the same stimuli were used in the estimation (Experiment 1) and the  
539 grouping task (Experiment 2), viewing conditions were different: peripheral viewing with limited  
540 presentation time (150 ms) in the estimation task and free viewing with unlimited presentation time in  
541 the grouping task. Hence, retinal stimulus locations and presentation time could have influenced the  
542 results in the two experiments. For example, different sets of discs could have appeared to group when  
543 viewed peripherally compared to when viewed freely. However, as proximity was the principal grouping  
544 factor, differences that would systematically reverse grouping strength of the same displays in the two  
545 experiments are implausible. Rather, proximity as a grouping factor should be stable and maintain the  
546 ordinal relationships among displays across eccentricities. Importantly, in the realm of contextual  
547 interactions, i.e., crowding, the very same effects of grouping (and ungrouping) have been observed in  
548 the fovea (Sayim et al., 2008; Sayim et al., 2010) and in the periphery (Manassi et al., 2012; Rosen &  
549 Pelli, 2015). Similarly, variations of presentation time should maintain the order of grouping strengths  
550 across displays (Haladjian & Mathy, 2015). Interestingly, investigations of grouping and ungrouping in a  
551 backward masking paradigm showed that complex Gestalts needed more time to yield ungrouping  
552 compared to basic features; however, presentation times were very short (20ms), and no modulation  
553 occurred beyond the presentation time in our Experiment 1 (150 ms, Sayim et al., 2014; see also  
554 Feldman, 2007; Kimchi, 1998). One possible explanation for the divergent numerosity estimation results  
555 of Experiment 1 and grouping results of Experiment 2 is that only single - or subsets of – grouped discs  
556 were sampled in a given trial in Experiment 1. As the number of (perceived) groups was larger in the  
557 *radial* compared to the *tangential* condition (Experiment 2), and therefore the average number of discs  
558 per group was smaller, numerosity estimates based on single (or a few) groups would be lower.  
559 However, given the frequent overestimation in the current study, it is unlikely that such sub-sampling

560 (without overcompensation) has occurred. Another factor that could underlie the diverging results in  
561 Experiments 1 and 2 is that different groups of observers participated in the two experiments. In recent  
562 experiments with similar stimuli (including the radial-tangential manipulation), we found similar results  
563 with a different group of observers (66 participants), providing further evidence that numerosity  
564 estimates depend on the (*radial* or *tangential*) arrangement of items. In Experiment 2 of the current  
565 study, 87% of the observers indicated more groups in the radial than in the *tangential* condition (on  
566 average for all numerosities), while only 13% showed the opposite pattern, indicating a robust pattern  
567 of results across participants. Hence, it is unlikely that a different group of observers would show the  
568 opposite pattern of results, i.e., higher numerosity estimates and a larger number of perceived groups in  
569 the radial condition compared to the tangential condition.

570 Overall, we demonstrated that numerosity perception was anisotropic in regard to *radial* versus  
571 *tangential* arrangements. We suggest that redundancy masking is one of the potential determining  
572 factors in numerosity estimation. Going beyond purely physical stimulus descriptions by taking into  
573 account asymmetries of the visual field in spatial vision will help to shed light on the underlying  
574 mechanisms of numerosity perception.

575

## Appendix

**Supplementary Table S1**

*A summary of physical properties for the radial and the tangential displays across all numerosity ranges.*

	Numerosity range 21-25		Numerosity range 31-35		Numerosity range 41-45		Numerosity range 49-53		Numerosity range 54-58	
	Tan(SD)	Rad(SD)	Tan(SD)	Rad(SD)	Tan(SD)	Rad(SD)	Tan(SD)	Rad(SD)	Tan(SD)	Rad(SD)
Average spacing (°)	6.80(0.07)	6.74(0.06)	7.93(0.07)	7.91(0.05)	9.16(0.09)	9.16(0.08)	10.41(0.09)	10.37(0.09)	11.44(0.09)	11.38(0.09)
Convex hull (°)	35.47(1.04)	36.57(0.98)	48.28(0.78)	48.49(0.67)	60.93(0.98)	61.49(0.96)	73.78(1.12)	74.29(0.96)	84.43(1.39)	85.38(0.98)
Average eccentricity (°)	5.07(0.05)	5.03(0.05)	5.88(0.05)	5.86(0.04)	6.71(0.07)	6.72(0.06)	7.54(0.07)	7.54(0.06)	8.21(0.06)	8.20(0.07)
Occupancy area (Convex hull 2D volume)	88.83(3.60)	90.81(3.22)	157.45(4.04)	156.30(4.05)	249.79(4.69)	251.38(5.06)	367.54(6.48)	368.95(6.79)	482.35(10.79)	485.45(8.17)
Density (item/deg <sup>2</sup> )	0.54(0.02)	0.52(0.01)	0.31(0.01)	0.30(0.01)	0.21(0.01)	0.21(0.01)	0.16(<0.01)	0.16(<0.01)	0.13(<0.01)	0.13(<0.01)

*Note.* Tan: Tangential displays; Rad: Radial displays. SD: Standard deviation. Convex hull and occupancy area were computed using the Qhull library (Barber et al., 1996) with Python. Density was calculated using the numerosity divided by occupancy area, excluding the empty central region (46.28 deg<sup>2</sup>).

1 **Supplementary Table S2**2 *Averaged radial alignment scores (RAs) for each numerosity range*

Numerosity range	Radial (SD)	Tangential (SD)
21-25	0.075(0.255)	0 (0)
31-35	0.466 (0.532)	0 (0)
41-45	1.378 (0.808)	0.049 (0.165)
49-53	2.939 (1.080)	0.525 (0.461)
54-58	3.378 (1.207)	1.447 (0.914)

3

4 **Supplementary Table S3**5 *Partial correlations (partial  $r_1$  and CI<sub>95%</sub>) between deviation scores (DVs) and radial alignment scores*6 *(RAs) controlling for numerosity and partial correlations (partial  $r_2$  and CI<sub>95%</sub>) between DVs and*7 *crowding strength controlling for numerosity*

Numerosity range	partial $r_1$	CI <sub>95%</sub>	partial $r_2$	CI <sub>95%</sub>
21-25	0.10	[-0.19 - 0.36]	-0.17	[-0.43 - 0.12]
31-35	-0.23	[-0.48 - 0.05]	-0.49***	[-0.68 - 0.25]
41-45	-0.31*	[-0.54 - 0.03]	-0.31*	[-0.54 - 0.03]
49-53	-0.52***	[-0.7 - 0.28]	-0.44**	[-0.64 - 0.18]
54-58	-0.50***	[-0.68 - 0.25]	-0.52***	[-0.7 - 0.28]
all	-0.40****	[-0.5 - 0.29]	-0.40****	[-0.5 - 0.29]

8 *Note.* \* $p < .05$ . \*\* $p < .005$ . \*\*\* $p < .001$ . \*\*\*\* $p < .0001$ 

9 In addition to circle sectors of 6°, we varied the size of the sectors from 1° to 12°, following the same  
 10 method as described above in Method. Too small and too large angles were expected to yield weaker  
 11 (or no) correlations with RAs as alignments would be rare (when angles were very small) or counted  
 12 when far beyond plausible interference zones (when angles were large). The results showed that this  
 13 was the case, with overall higher correlations for medium angle sizes (from about 5° to 9°).

14

15

16 **Supplementary Table S4**

17 *Descriptive Statistics: means and standard deviations of perceived groups in the tangential and the radial*  
 18 *condition for each numerosity range*

Numerosity range	Alignment condition	Mean (SD)
21-25	Tangential	6.13(2.50)
	Radial	7.37(2.66)
31-35	Tangential	9.3(3.66)
	Radial	10.7(4.72)
41-45	Tangential	12.0(4.74)
	Radial	13.6(5.72)
49-53	Tangential	13.9(5.95)
	Radial	15.1(6.91)
54-58	Tangential	15.7(7.80)
	Radial	18.0(6.48)

19

20

21



22 **Supplementary Table S5**

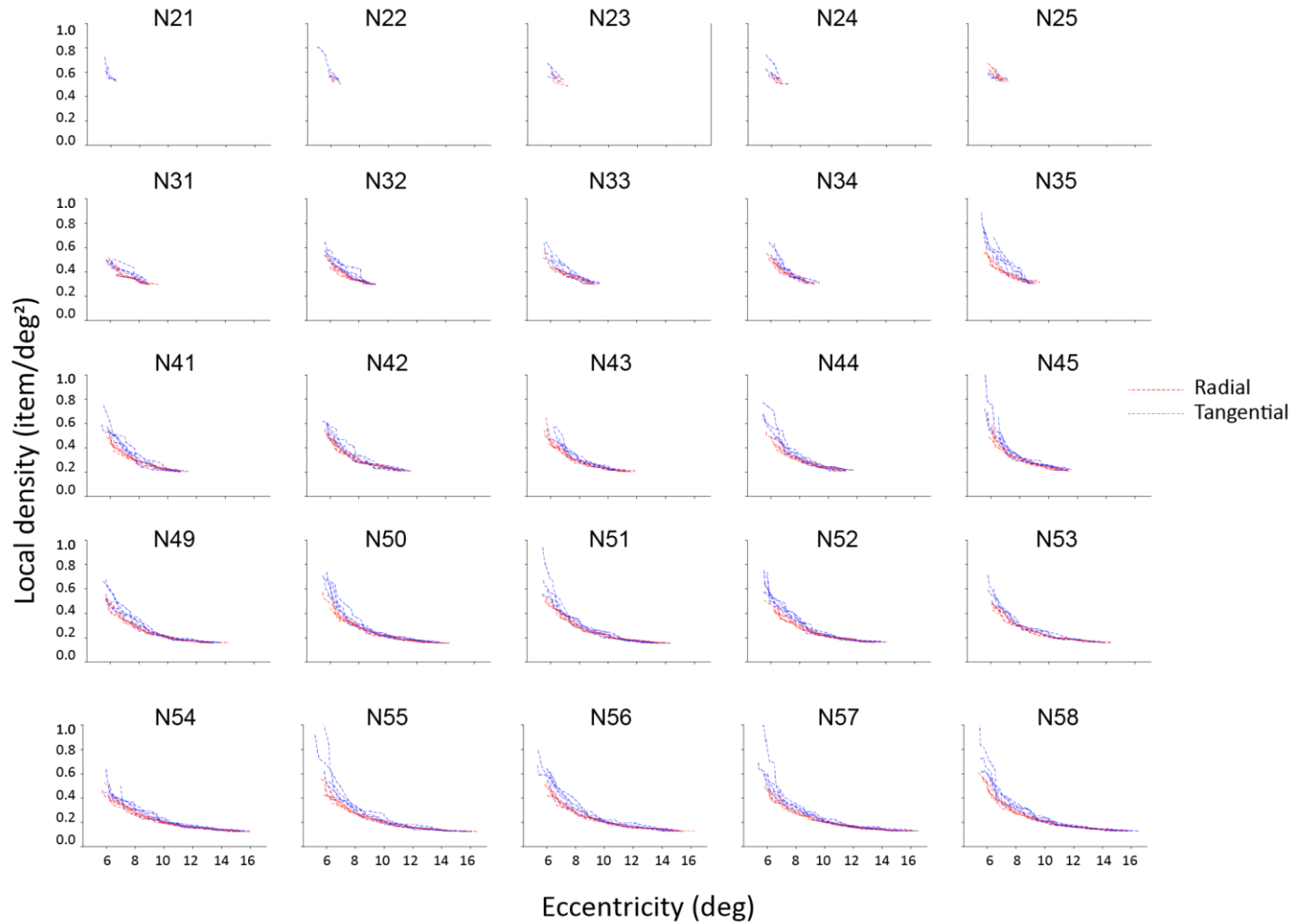
23 *Partial correlations between the number of perceived groups and deviation scores (DVs) controlling for*  
 24 *numerosity*

Numerosity range	Alignment condition	Partial r	CI95%
21-25	Tangential	0.18	[-0.25 – 0.54]
	Radial	-0.21	[-0.56 – 0.21]
31-35	Tangential	0.30	[-0.11 – 0.63]
	Radial	0.36	[-0.05 – 0.67]
41-45	Tangential	0.07	[-0.35 – 0.46]
	Radial	-0.22	[-0.57 – 0.2]
49-53	Tangential	0.34	[-0.07 – 0.65]
	Radial	0.02	[-0.38 – 0.42]
54-58	Tangential	-0.11	[-0.49 – 0.3]
	Radial	0.10	[-0.31 – 0.49]

25 *Note. All ps > .05*

26

27



28

29 **Supplementary Figure S1.** Local density as a function of eccentricity. Local density was measured using  
30 the number of discs of displays (that fall into the local convex hull region) divided by occupancy area,  
31 excluding the empty central region. Each curve represents the local density for a single display.

32

33

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