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Abstract. First in his paper «Utility Analysis and Interest» (1936) and then in two chapters of *The Pure Theory of Capital* (1941), Hayek develops a model of the interest rate that combines productivity and time-preference in a graphical framework inspired by Fisher 1930. Hayek claims that his model supports a productivity explanation of interest, in which time preference plays no role at all or only a minor role. We show in this paper that the arguments he puts forward in favor of a productivity explanation do not prove his point at all, and that his neglect of the role of time preference remains therefore unjustified. The consequences for his model of fully taking time preference into account are then investigated.[1]

Key words. Friedrich A. Hayek, interest rate, time preference, productivity

JEL classification. B25, E14

In two chapters of *The Pure Theory of Capital* (1941, Chap. XVII and XVIII), and originally in a 1936 paper on «Utility Analysis and Interest», Friedrich A. Hayek develops a graphical model that explains the determination of the interest rate in a simplified economic system that takes two elements into account, namely the intertemporal preferences and the productivity of capital. Even though his model is inspired by Fisher 1930, it possesses a number of specific characteristics. First, it is a macroeconomic model that aims at analyzing the most significant forces at work in an economic system considered as a whole. To this extent, this model belongs to the same category as Solow's basic macroeconomic model (Solow 1956). Second, Hayek leaves out entirely the phenomenon of lending and borrowing and focuses on

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productive investment only. Third, Hayek's model is conceived in such a way that intertemporal choices extend to an infinite number of future periods. The model is thus able to represent the step-by-step convergence process that leads the economic system towards a final stationary equilibrium.

One of the main questions that Hayek tries to answer with the help of his model is the question of the main determinant of the level of the equilibrium interest rate: is it productivity or time preference? He claims that *productivity is the main determinant* and offers two distinct arguments to that effect. His first argument is that marginal productivity is constant, and his second argument that marginal productivity is approximately constant when compared with marginal time preference. Now, in final equilibrium marginal productivity and marginal time preference are equalized. If marginal productivity is constant or approximately constant, then indeed the final equilibrium rate can be said to be "determined" by productivity alone in the sense that a change in marginal time preference has no effect, or only a very small effect, on this rate. The first argument (constancy of productivity) has already been criticized in Fillieule 2017 and will only be briefly mentioned below. The second argument (relative constancy of productivity when compared with time preference) will be the focus of this paper.

Section 1 presents the main version of Hayek's model, i.e., the version with a constant productivity. Section 2 analyzes and criticizes Hayek's argument on the relative constancy of productivity when compared to time preference. Finally, Section 3 examines the consequences for the model of getting rid of the constancy or relative constancy of productivity.

1. The initial model: constant marginal productivity

Hayek's model analyzes the determination of the interest rate in an economic system that produces a *homogenous consumption good* (or basket of consumption goods) and that is *controlled by a dictator* (or composed of a single actor). The latter faces a simple trade-off: more saving now (net saving and capital accumulation) allows for more production and consumption later, and less saving now (net dis-saving and capital decumulation) implies less production and consumption later. This productivity element is illustrated with the help of a *productivity curve* (Hayek calls it the "transformation curve") that relates the possible amounts of net saving (or dis-saving) to the larger (or smaller) amounts of consumption goods that will be available in the future. Hayek offers two successive versions of his model, the first one with a constant marginal productivity (linear productivity curve) and the second one with a diminishing marginal productivity (concave productivity curve). According to him, only the initial version with *constant marginal productivity* is consistent with the framework of his model, and it is therefore this version that is presented in this section. On the side of preferences, the intertemporal utility is illustrated with the help of a set of *indifference curves* determined by the dictator.

These two kinds of functions–productivity and utility–are represented in a framework that shows on the horizontal axis the current production of the consumption good, and on the vertical axis the production of this good *in each future period*. The main difference with the well-known Fisherian diagram is that in Hayek's graph the vertical axis shows the final product in each and every future period, not just in the next period. The illustrations, then, are straightforward:

- Figure 1a shows a stationary system in which the quantity Q_C of consumption goods is produced in the current period and in each future period.

- Figure 1b depicts a *linear* productivity curve used in the initial version of the model (no diminishing returns on capital accumulation); if an amount ΔS of net saving is invested *in the current period*, then the additional amount ΔQ will be produced *in each future period*; also, in this case, investment increases in the current period thanks to the net saving ΔS, and it will remain at this higher level in all future periods, sustaining the extra amount of consumption goods ΔQ produced per future period.
- Figure 1c shows a pattern of intertemporal indifference curves; these curves are convex, on account of the law of diminishing marginal utility.

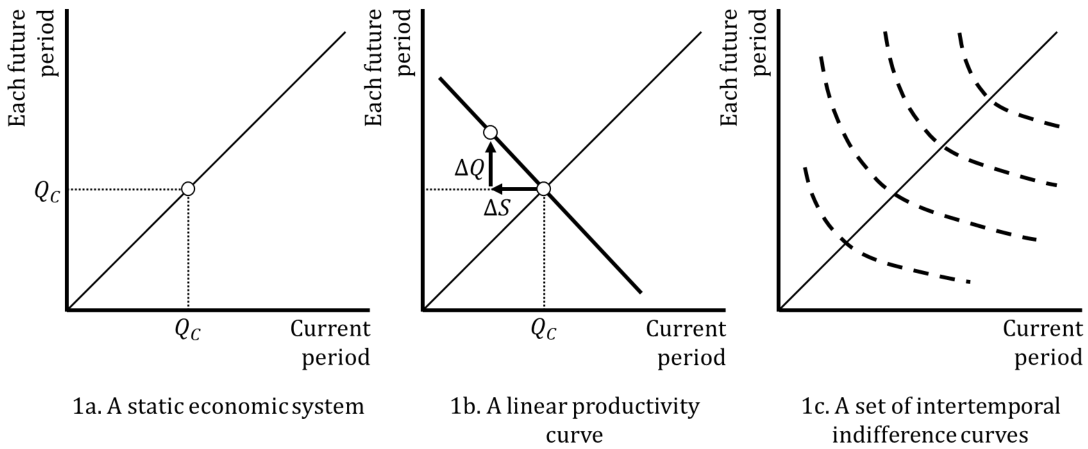


Figure 1. The Hayekian framework

In this framework, intertemporal choice takes place through a maximization of intertemporal utility. Figure 2a illustrates an optimal choice by the economic agent: saving the amount ΔS brings the economic system to the highest reachable indifference curve, i.e. to the greatest intertemporal utility. When the next period occurs, the actor is wealthier (higher up on the 45-degree line), and once again makes an optimal choice that maximizes intertemporal utility (Figure 2b). The economic system climbs along the 45-degee line because at each step of the process capital is accumulated, as the net saving of the current period will be part of the gross saving in the following periods. This choice process ends when the economic system reaches the point at which the productivity curve and an indifference curve are tangent to one another on the 45-degree line (Figure 2c). At this point, the actor cannot increase intertemporal utility anymore, whether through additional saving or through dis-saving: the economy is in final equilibrium. The existence of a final equilibrium requires here that marginal time preference increases with wealth (a purely empirical and quite implausible assumption): the higher the intertemporal curve, the steeper its slope on the 45-degree line.

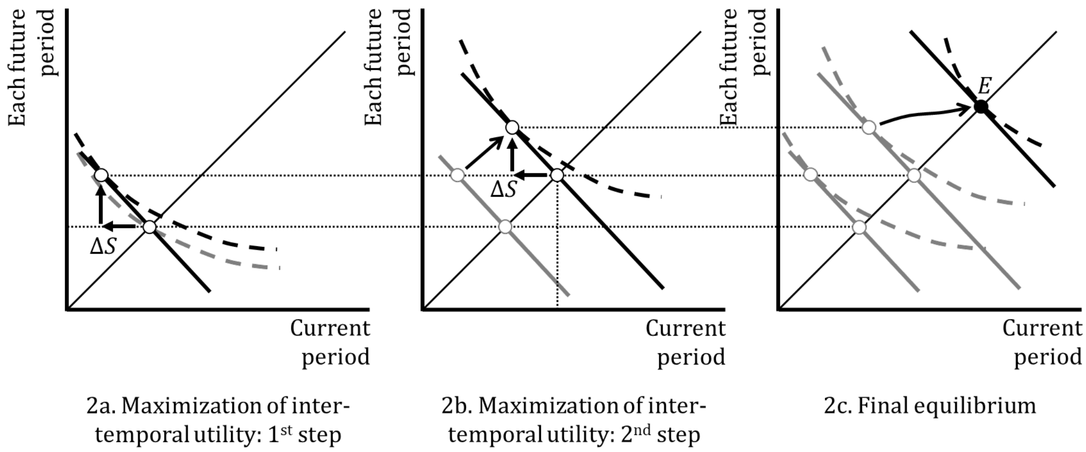


Figure 2. The process of convergence towards the final equilibrium

At each step of the convergence process, a corresponding interest rate appears. In the case of a constant marginal productivity, *this interest rate is one and the same during the process and in final equilibrium*: it is equal to the constant productivity rate. This result is demonstrated in the following way. At any given stage of the convergence process, the economic agent exchanges the amount ΔS of present consumption goods against an increase ΔQ of consumption goods in each future period:

$$- \Delta S \text{ in the present} \leftrightarrows + \Delta Q \text{ in each future period}$$

This intertemporal exchange implies an interest rate *r* that can be calculated thanks to the discount formula:

$$\Delta S = \frac{\Delta Q}{(1+r)} + \frac{\Delta Q}{(1+r)^2} + \frac{\Delta Q}{(1+r)^3} + \frac{\Delta Q}{(1+r)^4} + \cdots$$

This equation can also be written:

$$\Delta S = \Delta Q \left[\frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \frac{1}{(1+r)^4} + \cdots\right]$$

With an infinite number of future periods, the "perpetuity formula" can be used:

$$\sum_{n=1}^{\infty} \frac{1}{(1+r)^n} = \frac{1}{r}$$

It follows that:

$$\Delta S = \Delta Q \left(\frac{1}{r}\right)$$

The interest rate is:

$$r = \frac{\Delta Q}{\Delta S}$$

Since the productivity curve is a line with a constant slope, the ratio ($\Delta Q \div \Delta S$) is always the same–whether in or out of final equilibrium (the final equilibrium, however, is stationary while the intermediate equilibria are not). The interest rate r is therefore indeed a constant throughout the process, equal to the slope of this line (strictly speaking to the absolute value of the slope).

2. A critique of Hayek's arguments on the constancy or relative constancy of marginal productivity

It was shown in Section 1 that, if the productivity of capital is constant, then the interest rate is equal to this constant, whether in or out of final equilibrium. It can then be claimed that the level of the interest rate is "determined" (1941, 227) by the productivity of investment, since time preference has no effect whatsoever upon this level. *With a constant productivity, the shape of the pattern of intertemporal indifference curves cannot change the level of the final interest rate, and only impacts the path towards the final equilibrium and the extent of the capital accumulation in final equilibrium*. A lower time preference (less steep indifference curves) implies a larger amount of saving, a more capitalistic structure, and a greater production in final equilibrium (Figure 3a). A higher time preference (steeper indifference curves) has the opposite implications: a smaller amount of saving, a less capitalistic structure, and a smaller production in final equilibrium (Figure 3b). In both cases, however, the equilibrium interest rate is the same, as it is equal to the constant marginal productivity.

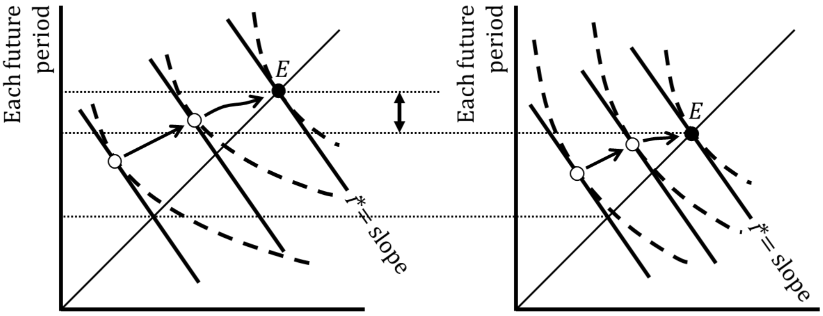


Figure 3. With a constant marginal productivity, i.e., a linear productivity curve, the pattern of indifference curves impacts the path towards equilibrium, but the level of the interest rate is constant (equal to the slope of the productivity line)

The argumentation by Hayek in favor of the productivity explanation of the level of the interest rate is based upon the constancy or relative constancy of the productivity of capital. We argue, however, that his two main arguments supporting this constancy are highly questionable. His first argument has already been dealt with by Fillieule 2017. Hayek claims that *in the framework of the model*, the productivity curve is *necessarily* linear: "the only assumption compatible with the case where there is only one commodity and only one possible investment period–is that up to a definite limit investments yield constant returns" (1941, 225). However, the diminishing returns on a factor of a productive process originate in the existence of another factor whose supply is fixed. The facts that there is "only one commodity" and "only one possible investment period" are totally irrelevant to the occurrence of the diminishing returns.

In his second argument, put forward in the next chapter of the book (1941, Chap. XVIII), Hayek gets "to the more realistic case where successive investments will yield decreasing returns" (1941, 229). He maintains, though, that the results of his Chapter XVII–obtained under the assumption of a constant marginal productivity–are still useful and relevant, because "*as compared with time preference*, the productivity of investment is likely to be *fairly constant*" (1941, 229, emphases added). If marginal productivity is "fairly constant" when compared to marginal time preference, then his conclusion remains valid that the interest rate is essentially determined by productivity.

Now, why is marginal productivity approximately constant relatively to time preference? Hayek's argument goes as follows. The curvatures respectively of the productivity curve and of the indifference curves do not change in the same way when the duration of the reference time period–the investment period–changes. Marginal productivity is the same whether the period is long or short: one more unit of saving raises future production of the same amount, independently of the length of the period during which the saving is done. It is quite different with time preference: the shorter the period, the more intense the time preference (the steeper the indifference curve). As Hayek puts it, "For a person with an annual income of £600, for example, saving £25 in the course of a year is an altogether different proposition from saving the same amount in the course of a month" (1941, 231). In other words, when the period shrinks, a given amount of saving represents a larger percentage of income, and the reluctance to save increases–i.e., the marginal time preference rises. Hayek then argues: "it is these relatively short periods which we have to consider if we want to understand the position of a person at different points on the path of saving" (1941, 232). So, a short time period should be considered, which leaves the marginal productivity constant while increasing greatly the marginal time preference. With this reasoning, he concludes that marginal productivity is approximately constant when compared with marginal time preference. This whole line of argumentation, however, is debatable and should in our opinion be rejected.

(1) Hayek explains that, if shorter periods of time are considered, then marginal time preference will increase while marginal productivity will remain the same, hence the

constancy of the productivity. The problem with this justification of constancy is that it is irrelevant to the issue at hand. The kind of constancy that is required (in order to explain the level of the interest rate by productivity alone) is a constancy of marginal productivity *at the different levels of wealth*. Figure 4a illustrates a configuration in which the marginal productivity, i.e. the slope of the productivity curves across the 45-degree line, is constant. In this configuration, indeed, the final equilibrium interest rate, if it exists, is necessarily equal to the constant productivity of investment. Now, the constancy that Hayek is talking about in his argument is of an entirely different kind. It is illustrated in Figure 4b: *at a given level of wealth* and with a given marginal unit of saving, a shortening of the time period (i) raises marginal time preference and (ii) leaves marginal productivity untouched. This reasoning only applies at a given point on the 45-degree line. It does not say anything about the level of marginal productivity at other points on this line, and therefore does not show that the marginal productivity is "fairly constant" across the board. The conclusion that productivity is the sole or main determining factor of the level of the interest rate does not follow.

(2) Hayek claims that we have to consider "relatively short periods" in order to analyze the "path of saving" of an individual. This assertion is debatable because the time period used in the model is the period of production *of the economic system*. This period cannot be shortened on the ground that the focus is on a specific individual rather than on the whole society.

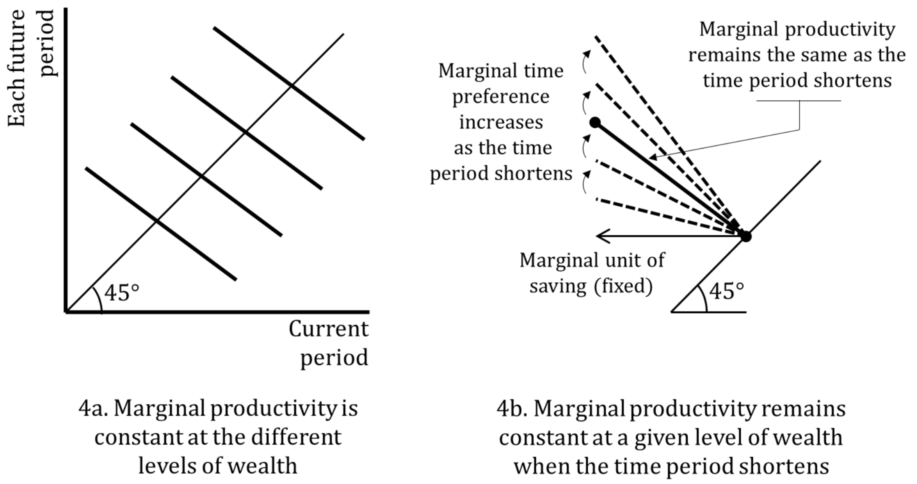


Figure 4. Constancy of productivity at different levels of wealth with a given time period (4a) and constancy of productivity at a given level of wealth with different time periods (4b)

(3) Hayek's argument suffers from yet another problem. It presupposes that the unit of saving remains the same while the duration of the time period diminishes. But would not the relevant unit of saving depend on the time period considered by the actor? In Hayek's example, the individual *annually* saves £25 out of £600. Hayek then supposes that the period is shortened from one year to *one month*, while keeping the amount of saving at £25. But in this case, the individual would not consider a *monthly* saving of £25. Rather, the corresponding monthly saving envisioned by the actor would be (\$25 ÷ 12) out of a monthly income of (£600 ÷ 12). In other words, the unit of saving would be proportional to the length of the period considered. Under these circumstances, the marginal rate of time preference does not necessarily increase with the shortening of the time period. Would the actor be more reluctant to save monthly (\$25 ÷ 12) out of a monthly income of (£600 ÷ 12) than to save annually £25 out of an annual income of £600? Not necessarily. Hayek's numerical illustration rests upon a questionable assumption, namely that the unit of saving remains the same while the time period shrinks.

(4) In an appendix at the end of his book (1941, 413-423), Hayek goes back to the topic of the determination of the interest rate by productivity or time preference. However, he does not offer here any new or additional argument and just repeats the conclusion that he had reached in his Chapters XVII and XVIII, namely that the level of the interest rate is determined by productivity only:

> [T]ime preference is a subordinate factor compared with the productivity of investment in determining the rate of interest, since it operates only by way of determining the rate of saving and the rate of capital accumulation, and hence the productivity of investment. In the short run, it merely adapts itself to the given marginal productivity of investment. (1941, 413)

His defense of productivity as the sole or main determinant of the interest rate is not reinforced at all in this appendix.

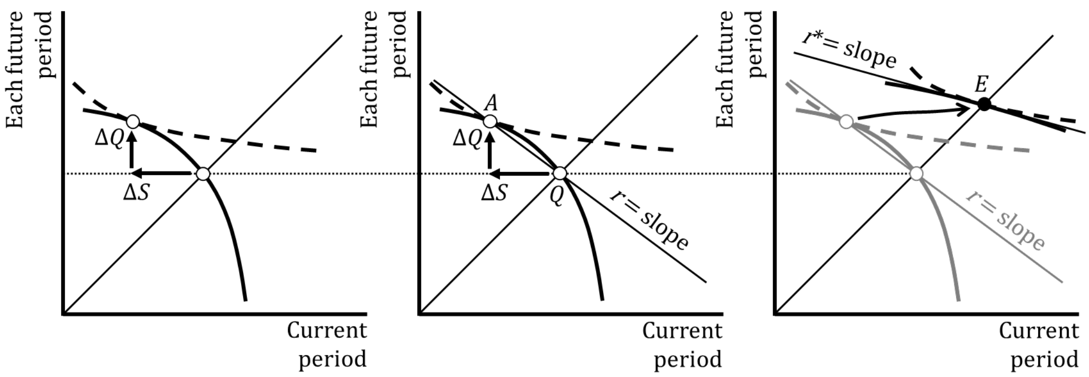
To sum up, none of the arguments put forward by Hayek is convincing. *The rejection of his arguments does not prove that his hypothesis of the constancy (or near constancy) of productivity is wrong, but as far as we are concerned we do not see any theoretical or empirical reason to suppose that marginal productivity is more rigid than marginal time preference.* We therefore put the constancy hypothesis aside and are now going to assess the consequences that follow for the model.

3. Consequences of the rejection of the constancy-of-productivity hypothesis

When the constancy or relative constancy of productivity is rejected, the major consequence is that the productivity-only explanation of interest becomes untenable. *Both time preference and productivity impact the level of the final equilibrium interest rate*. Two other more subtle consequences follow, regarding the interest rates during the convergence process and the existence of a final equilibrium.

3.1 The interest rates during the convergence process

With constant returns, the rate of interest at each step of the process and in final equilibrium is a constant. It is equal to the constant productivity of capital or alternatively to the constant slope of the productivity line (see Section 1). This result no longer applies when returns are diminishing, i.e., when the productivity curve is concave instead of linear. At a given step of the process, the interest rate is calculated with the same formula, namely the ratio ($\Delta Q \div \Delta S$) at the point that maximizes intertemporal utility. However, *this ratio is no longer a constant*. Figure 5 illustrates the optimum point at a step of the process (Figure 5a), the interest rate *r* as the slope of the line joining the point on the 45-degree line to the temporary optimum (Figure 5b), and the final equilibrium rate *r** (Figure 5c). It can be observed on this figure that the interest rate changes throughout the convergence process: it diminishes until it reaches its minimum value as the final equilibrium rate.



5a. An optimal choice (system out of final equilibrium) 5b. The interest rate $r = \Delta Q/\Delta S$ is the slope of the line 5c. The interest rate r^* in final equilibrium

Figure 5. With a concave productivity function, the interest rate is not a constant during the convergence process: it diminishes towards its final equilibrium value

3.2 The existence of a final equilibrium: increasing vs. decreasing "impatience"

In Hayek's initial model with constant returns, in order for a convergence process to take place and a final equilibrium to exist, *marginal time preference must increase with wealth*. Graphically, the slopes of the indifference curves across the 45-degree line must become steeper as the curves are higher in the quadrant. This requirement clearly appears on Figure 2: the process of capital accumulation is launched because at the starting point the marginal productivity is above the marginal time preference, and since the former is a constant, the marginal time preference has to increase in order to catch up and eventually become equal to the marginal productivity in final equilibrium. Molavi Vasséi 2015 analyzes in depth this "increasing impatience," as he calls it. Now, when the returns on investment are decreasing, the picture changes: for a process to occur and a final equilibrium to exist, *marginal time preference can either increase, decrease, or remain constant with wealth*. Figure 6 illustrates these three cases with a simplified representation that displays, not the whole curves, but the marginal values only (hence the straight lines). In each of the three graphs, the pattern of diminishing marginal productivity is the same and only the pattern of time preference changes. In Figure 6a the marginal time preference increases, and the final equilibrium is reached quicker and at a lower point. In Figure 6b the marginal time preference decreases, and the final equilibrium is reached later and at a higher point. In Figure 6c, with a constant time preference, the final equilibrium is reached at an intermediary point.

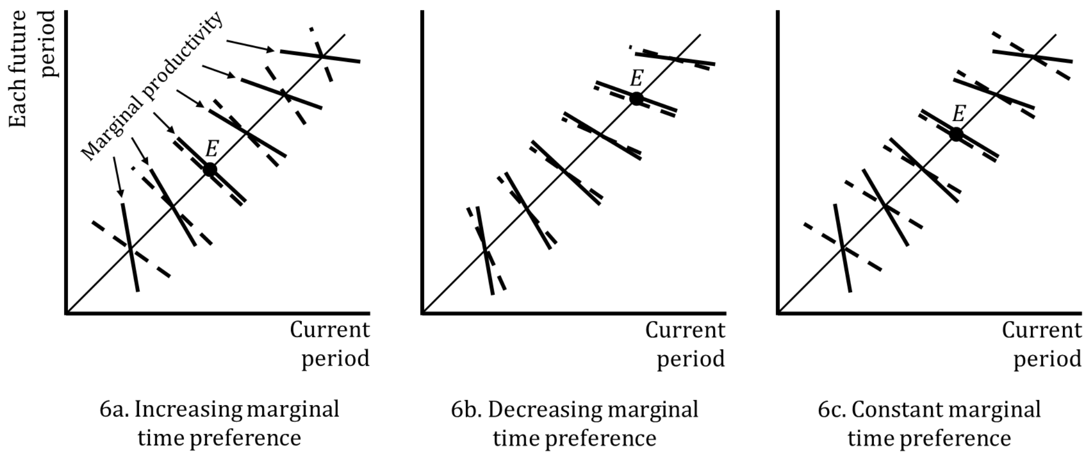


Figure 6. The final equilibrium *E* when marginal time preference respectively increases (6a), decreases (6b), and is constant (6c) with wealth; the pattern of marginal productivity is the same in all three graphs

When wealth grows from a capital accumulation, the returns necessarily diminish (everything else equal). On the side of time preference, on the other hand, there is no such necessary relationship (Block et al. 2006). As illustrated respectively in Figures 6a, 6b, and 6c, the relationship between wealth and time preference can be positive (increasing "impatience"), negative (decreasing "impatience"), or constant. If "impatience" increases or remains constant while productivity diminishes, a final equilibrium necessarily exists since, at some point, the two marginal rates will cross path. However, the most likely case in the real world is *decreasing* "impatience." It is indeed reasonable to suppose that, in general, as people become wealthier, they are less and less reluctant to save and invest a given amount of present wealth. Now, if productivity and "impatience" *both decline* and at the beginning the former is above the latter, then *the existence a final equilibrium requires that productivity falls quicker than "impatience,"* so that the two marginal rates eventually intersect. This is the configuration illustrated in Figure 6b. If, on the other hand, marginal productivity falls at the same pace or slower than "impatience" (and both remain strictly positive), then the former will never catch up with the latter: no final equilibrium will occur, and capital accumulation will keep on forever.

4. The empirical nature of Hayek's argumentation

It follows from the previous analysis that Hayek's argumentation in favor of productivity is *empirical*, not logical. From a logical viewpoint, his model is compatible with a time-preference explanation as well as with a productivity explanation of the interest rate. Figure 6c above illustrates his model in the case of a *constant marginal time preference* (facing a diminishing marginal productivity). The slopes of the indifference curves across the 45-degree line have all the same value. In this configuration, the final interest rate is "determined" by time preference alone since it is necessarily equal to the constant slope of the indifference curves: marginal time preference is the sole determinant of the level of the final interest rate, while marginal productivity explains the extent of the process and the level of capital accumulation. This case is symmetrical to the main one displayed in Hayek's book, with a constant marginal productivity and a variable rate of time preference (illustrated in Figures 2 and 3). Hayek is perfectly aware that there are two polar cases, one with a constant productivity (determination of the interest rate by productivity) and the other with a constant rate of time preference (determination of the interest rate by time preference). He offers an illustration of these two patterns, not in his Chapters XVII and XVIII but in the appendix on "Time Preference and Productivity" (1941, 422-423). Since the logical structure of his model is neutral between the time-preference and the productivity explanations, he must indeed resort to empirical arguments to justify his pick of the productivity side.

It is relevant in this context to note that Hayek was already leaning towards the productivity explanation of the interest rate long before his 1936 paper. In a paper published almost a decade earlier, titled «On the Problem of Interest Theory» (2015 [1927]), he explains that time preference is not at all required to explain the existence of interest, and that productivity of capital alone suffices. There, he also criticizes time preference (that he defines as "the undervaluation of future needs") as an "*ad hoc* assumption" that is "contradicted by the facts" (2015 [1927], 20-21), although he does not specify any of these "facts." It appears, therefore, that when Hayek developed his model of the interest rate in the mid-1930s, he had already formed an opinion on its determination many years ago. The empirical arguments used in 1936 and 1941 rationalize his enduring belief that the time-preference explanation of the interest rate should be rejected in favor of the productivity explanation. In his last paper devoted to the topic of interest theory published in 1945, Hayek reiterates his empirical argumentation and studies in more detail the case when capital accumulation falls short of its expected level, which changes the shape of the productivity curve and therefore impacts the respective roles of productivity and time preference.

Conclusion

This paper may at times seem overly critical of Hayek, so we should make it clear that his model is in our opinion quite brilliant, both from a theoretical and a pedagogical viewpoint. The interplay between the subjective and objective elements in the decision to save and invest is set in an elegant framework that usefully clarifies the logic of capital accumulation. However, Hayek's interpretation impoverishes and even to some extent betrays his own model. By maintaining that marginal productivity is constant or "fairly constant," he offers a one-sided explanation of interest. His arguments on the constancy of productivity are so weak that one wonders how he could ever find them convincing. It is likely that, from the start, he *wanted* to come up with a productivity explanation of the interest rate in the tradition of Böhm-Bawerk 1959 [1889] and Wicksell 1970 [1893]. Hayek 1936, 1941 is indeed the last major economist of the Austrian School to claim that productivity is the main determinant of the equilibrium interest rate (Mises 1998 [1949], Rothbard 1962, and Kirzner 1993 all support a time preference theory). It turns out, however, that when the diminishing returns on capital accumulation are properly taken into account, Hayek's model does not depart from the standard Fisherian viewpoint of a determination of the interest rate by both productivity and time preference.

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