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This paper aims to propose an Improved Starting Point (ISP-) Newton method applied to vector potential $\boldsymbol{A}$ formulation for circuit coupled magnetostatic problems. These problems usually are known to vary greatly during the time. This impacts the quality of the initial guess of the Newton method and thus increases significantly the computational cost. The Newton method with vector potential formulation A has been analyzed. Numerical examples show the performance of our proposed ISP-Newton method.

Index Terms—**Convergence, Newton method, improved starting point, coupled magnetic-electric circuits problems.**

I. Introduction

Nonlinear magnetostatic problems are widely encountered in electrical engineering. The computation of the magnetic fields requires the resolution of nonlinear problems due to the saturation of ferromagnetic materials. With the finite element method (FEM), these nonlinear problems are widely solved by two techniques, namely the fixed point method and the Newton method. The fixed point method consists in splitting the nonlinear term into two parts, respectively the linear term and the nonlinear residual term. The convergence can only be achieved under the condition that the linear term is uniformly continuous and strongly monotone [1]. And, unfortunately, its convergence rate can be very slow, as reported in [2]. Thus, the fixed point method may introduce an unacceptable computational time.

The Newton method is based on a 1st order Taylor expansion of the nonlinear term. It converges locally at a quadratic rate [3]. Moreover, the stiffness and Jacobian matrix should be evaluated at each Newton iteration. In the case of strong non-linearity, the Newton method may fail to converge. That's why line search techniques are widely applied to make the convergence process more stable [4]. However, with the relaxation, the convergence speed could not be fast, since the determination of the optimum value is too expensive in CPU time [5]. Some other authors developed the line search techniques in order to predict the Newton step. For example, in [6], the Newton step is approximated by a linear interpolation using two points of the derivative of the energy functional. It should be mentioned that the obtained step size is not an exact solution. Other approaches reported in [7], an exact functional minimization is considered for improving the nonlinear convergence behaviour. Unfortunately, this may introduce more computational burden. e In [8], a coupling way of the fixed point and the Newton method has been introduced. The idea is to use the fixed point method to provide a starting value, to be used in the Newton scheme later. This can reduce the number of iterations and improve the robustness only in the case with high saturation.

Because of this only local convergence property, starting with a good initial solution can improve the robustness and the speed of the method. Finding a priori information of the exact solution becomes useful. Recently, [9] reported a well-posed condition and proposed an effective particular initial function for 2D p-Laplace problem. Following this idea, in this work, we propose an improved initial point for the Newton method applied to magnetostatic problems with the vector potential $\boldsymbol{A}$ formulation when variations are important. The paper is organized as follows: in Section 2, the context of the problem is presented with the different methods. Section 3 constitutes the main numerical part of this work, in order to evaluate the performance of the proposed ISP-Newton method. Finally, a conclusion is given in Section 4.

II. Numerical Modelling of non-linear magnetostatic problems

Let us consider an open, connected domain $\Omega \subset \mathbb{R}^3$ with a Lipschitz boundary $\Gamma = \partial\Omega$. Given a divergence-free applied source current density $\mathbf{J}_s$, the magnetic flux density $\mathbf{B}$ and the magnetic field $\mathbf{H}$ verify the following equation

$$\boldsymbol{div}\ \boldsymbol{B} = \boldsymbol{0} \text{ and } \boldsymbol{curl}\ \boldsymbol{H} = \boldsymbol{J}_s. \tag{1}$$

The nonlinear relationship of magnetic materials reads:

$$\boldsymbol{H} = \nu(\boldsymbol{B})\boldsymbol{B}, \tag{2}$$

where $\nu(\boldsymbol{B})$ presents the reluctivity. By introducing the magnetic vector potential $\boldsymbol{A}$ which satisfies $\boldsymbol{B} = \boldsymbol{curl}\ \boldsymbol{A}$, we obtain the following vector potential formulation:

$$\boldsymbol{curl}\ (\nu\ \boldsymbol{curl}\ \boldsymbol{A}) = \boldsymbol{J}_s. \tag{3}$$

To impose the voltage, we consider the coupling with the external circuit [10]. Thus, the following equation is added to the system (3):

$$U = RI + \frac{d\phi}{dt} \text{ with } \phi = \int_D \boldsymbol{A}.\boldsymbol{N} dv. \tag{4}$$

Solving these with a FEM solver involves updating the current solution $\boldsymbol{A}_n$ with the so-called Newton step:

$$\boldsymbol{A}_{n+1} = \boldsymbol{A}_n - \lambda DF(\boldsymbol{A}_n)^{-1}F(\boldsymbol{A}_n), \tag{5}$$

where $F(\boldsymbol{A})$ is the function computed with FEM, DF its derivative and $\lambda \in [0,1]$ is a step reduction factor computed with a line search algorithm to ensure convergence of the Newton algorithm.

When solving this equation for a time dependent problem, we have a natural initial guess $\boldsymbol{A}_0$ to start the algorithm. Unfortunately, when variations are high as it is very common in coupled circuits problems, this starting point may not be close enough and the line-search is mandatory to ensure the convergence and the computations might be slowed down with many step reductions. We propose to refine it with two successive linearisations which could be seen as two steps of a fixed-point solver:

$$\boldsymbol{curl}\ (\nu_{\boldsymbol{A}_0}\boldsymbol{curl}\ \boldsymbol{A}_1) = \mathbf{J}_{\mathrm{s}} \text{ and } \boldsymbol{curl}\ (\nu_{\boldsymbol{A}_1}\boldsymbol{curl}\ \boldsymbol{A}_2) = \mathbf{J}_{\mathrm{s}}. \tag{6}$$

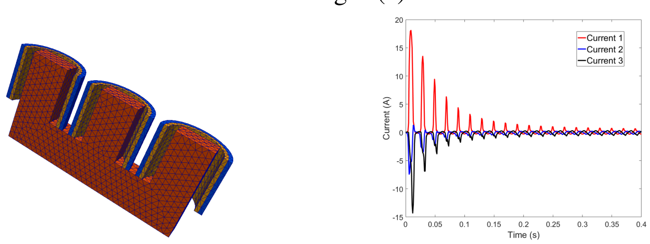
These two additional steps are inspired by the ISP-Newton algorithm [11]. As states in [9], one linearisation is not enough and iterations after the second linearisation are less efficient than a usual Newton step, as we can see in the numerical results in the next section.

III. Industrial example of a three-phase transformer

	Newton	ISP-Newton			Fixed Point
Step	$\boldsymbol{A}_0$	$\boldsymbol{A}_1$	$\boldsymbol{A}_2$	$\boldsymbol{A}_3$	$\boldsymbol{A}_\infty$
1	4	6	2	3	17
2	7	11	2	3	21
3	9	13(**1**)	2	3	23
4	13(**1**)	15(**2**)	6	9	27
5	15(**7**)	18(**9**)	9(**2**)	11(**2**)	33
6	12(**4**)	16(**5**)	8(**4**)	9(**2**)	34
7	12(**6**)	15(**5**)	7(**2**)	9(**1**)	36
8	12(**5**)	14(**7**)	8(**1**)	9(**1**)	34
9	12(**4**)	14(**5**)	7(**1**)	8(**1**)	30
10	14(**4**)	16(**2**)	8(**2**)	9(**1**)	33
Iterations	110(**31**)	138(**36**)	59(**11**)	73(**8**)	292
CPU	7m26s	9m11s	4m53s	5m47s	13m21s

TABLE I
COMPUTATIONAL RESULTS FOR THE THREE-PHASE TRANSFORMER.

To validate our method on industrial case, we propose to deal with a strongly nonlinear problem coupled with electrical circuits. This involves powering up a three-phase transformer at no load. Due to symmetric consideration, only one quarter of the structure is modelled. The used mesh holds 29855 elements and 6406 nodes as shown in Fig. 1(a).



(a) Structure (b) Currents

Fig. 1. Mesh for the quarter of the three-phase transformer and primary currents in the three phases.

Three primary currents and three-phase sinusoidal voltages have been computed by each method to compare the efficiency of our approach. The frequency has been taken as $f_0 = 50Hz$. The problem is solved with the 3D Code_Carmel[1] on 2 period $T = 1ms$ and we consider a uniformly time discretization which has 40 steps for each time step. A classical large rush currents due to the high saturation of the magnetic core are obtained as shown in Fig. 1(b).

This problem has been evaluated with different schemes, the Newton method, the ISP-Newton method and the fixed point starting from different points as detailed in Tab. I. The number of Newton iterations as well as the number of corrections in the line search method are giving for different schemes starting from the previous solution of the classical Newton point $\boldsymbol{A}_0$, the first step of our ISP-algorithm named $\boldsymbol{A}_1$, the proposed starting point $\boldsymbol{A}_2$, the starting point $\boldsymbol{A}_3$ that it can be found after two linearisations, or with $\boldsymbol{A}_\infty$ after doing an infinity of linearisations. In the last two lines, the total number of the nonlinear iterations, the corrections in the line search method (the value between brackets) and the computational cost are given.

As theoretically expected, the fixed point requires the higher cost in term of number of iterations and computational time. However, with our ISP-Newton method, it appears that rather important computational savings in terms of Newton iterations, as well as corrections during the line search are achieved for the whole simulation. Moreover, a speed-up from 1.20 to 2.01 can be obtained comparing with the other starting points.

IV. Conclusion

In this paper, an ISP-Newton method is proposed for the vector potential formulation in 3D nonlinear coupled circuit problems. The implementation of our proposed method is relatively easy, since only two linear systems have to be constructed and solved. The validation was performed on a three phases transformer and the numerical results show an improvement of the robustness and the convergence rate for different solvers. Moreover, the number of Newton iterations, as well as the computational time, are both reduced.

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[1] See https://code-carmel.univ-lille.fr